


[Home](#)
[Welcome Address](#)
[Organizations](#)
[Committees](#)
[General Information](#)
[Plenary Speeches](#)
[Programme](#)
[Author Index](#)
[Search](#)

*Proceedings of*  
**2014 IEEE Chinese Guidance, Navigation and Control Conference**  
**IEEE CGNCC 2014**

**8 – 10 August 2014**  
**Yantai, China**

Sponsored by



Co-Sponsored by



Technical Co-Sponsored by



Organized by



IEEE Catalog Number: CFP1429X-CDR

ISBN: 978-1-4799-4404-0

Powered by: Beijing Leading Innovation Technology Development Co.,Ltd

**Copyright and Reprint Permission:**

Abstracting is permitted with credit to the source. Libraries are permitted to photocopy beyond the limit of U.S. copyright law for private use of patrons those articles in this volume that carry a code at the bottom of the first page, provided the per-copy fee indicated in the code is paid through Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923. For reprint or republication permission, email to IEEE Copyrights Manager at [pubs-permissions@ieee.org](mailto:pubs-permissions@ieee.org). All rights reserved. Copyright ©2014 by IEEE.

	0526	SatB8.3	Hao Fei	0592	SatB11.46
Gong Huajun	0308	SunB10.2		0595	SatB6.9
Gong Jian	0648	SunB2.1	Hao Qishi	0400	SatA10.5
Gong Ligang	0081	SatA4.3	Hao Ran	0680	SatA10.7
Gong Qinghai	0466	SunB7.8	He Ao	0191	SatB11.38
Grzegorzewski Marek	0294	SunB2.5	He Bi-hai	0129	SunB11.12
Gu Chao	0083	SunB3.4	He Bing	0335	SatB10.3
Gu Tianyuan	0383	SunA11.31	He Chao	0191	SatB11.38
	0386	SunB11.28	He Chaofan	0692	SunB7.4
Guan Changbin	0381	SatB3.3	He Jiang-liang	0688	SunA11.18
Guo Hongwu	0289	SatB9.3	He Jianlun	0635	SunA11.36
Guo Jia	0459	SunA5.4	He Li	0232	SunA11.15
Guo Jian	0577	SunA11.52	He Ping	0510	SunA5.1
Guo Jiao	0244	SatB8.5	He Shaoming	0046	SunB1.2
Guo Jun	0198	SatA4.7	He Xiang	0522	SunB8.2
Guo Lei	0095	SunB1.8		0524	SunA3.7
	0103	SunB5.5		0528	SunA3.8
	0151	SatB11.15	He Xiaofeng	0544	SatB11.21
	0375	SunA11.10	He Zhen	0457	SatA3.2
	0579	SatA11.35	Hei Wen-Jing	0191	SatB11.38
	0590	SunA10.6	Hong Guang	0487	SunB11.37
Guo Lili	0641	SunB7.3	Hong Wei	0698	SatB8.8
Guo Linliang	0314	SatB11.31	Hou Jian	0198	SatA4.7
Guo Meifeng	0227	SatA9.7	Hou Manyi	0543	SatB6.2
	0359	SatB11.17	Hou Man-Yi.	0454	SunA5.3
Guo Pu	0497	SunA7.7	Hou Zhongxi	0247	SatB1.8
Guo Xiao	0193	SatB3.4	Hu Caibo	0259	SunA11.44
Guo Yang	0629	SatB10.6		0298	SunA7.2
Guo Yingxin	0045	SatA4.8	Hu Changhua	0480	SatB10.5
Guo Zhenyun	0168	SunB5.7	Hu Chaofang	0620	SatA11.25
Guo Zhixin	0701	SatB7.2	Hu Chunhe	0442	SunA6.4
			Hu Dandan	0598	SatA8.1
				0601	SatA8.2
				0611	SatB11.26
Han Cunwu	0567	SatA6.4		0611	SatB11.26
Han Hui-Lian	0375	SunA11.10		0317	SatB7.3
Han Li	0074	SunB5.2	Hu Jiwen	0444	SunA5.2
Han Liang	0520	SatA8.5		0103	SunB5.5
Han Pengxin	0551	SatB3.9	Hu Qinglei	0311	SatA6.1
Han Songshan	0562	SatA6.3	Hu Rong	0302	SatA11.32
Han Xiao-Jun	0324	SunA11.9	Hu Shaolin	0643	SatB11.7
Han Yang	0289	SatB9.3		0106	SunB3.5
Han Yingchun	0467	SatA5.8	Hu Songjie	0189	SatA11.42
Han Yongqiang	0482	SatB1.3	Hu Wunong	0190	SunA4.7
Hang Dongze	0428	SunB11.32			

**H**

# Linear Parameter-Varying Attitude Controller Design for a Reusable Launch Vehicle during Reentry

He Chaofan, Yang Lingyu, Wang Zhenchao, Sun Bin, Zhang Jing

**Abstract**—Considering the perturbation within a wide range of flight parameter (such as height, mach), and the strong uncertainty of aerodynamic and atmospheric parameter, a LPV(Linear Parameter-Varying) control method is studied for a RLV(Reusable Launch Vehicle) during reentry, which can make the controller designed envelope-oriented and obtain the self-scheduled ability. A standard LPV controller design process is given first, and the problem of LPV controller's large data volume is studied especially, for it will perform an exponential growth with the increase of order and number of LPV vertex models. Then the longitudinal/lateral controller separate design method based on coupling analysis and the LPV vertex models construct method based on model characteristics analysis are introduced, to decrease the data volume and make the LPV controller more applicable. At last, a LPV controller is designed for some RLV during reentry using the methods introduced above, and the 6-degree nonlinear simulation results demonstrate that the methods can reduce the data volume remarkably, while guarantee the controller's adaptability, tracking performance and robustness good enough.

## I. INTRODUCTION

The reentry phase is the most challenging phase of RLVs. During reentry, there exist the strong uncertainty of aerodynamic and atmospheric parameter, as well as the complex circumstance concluding electromagnetic interference and wind disturbance and so on, which means the necessity of strong robustness of RLVs' controller to have the ability to keep stable under different kinds of uncertain and disturbance situation. Besides, the flight parameters, such as height and mach, change within a large range, which results in the obvious difference of aerodynamic characteristics and strong coupling. So the workload of design of controller based on traditional gain scheduling strategy will be beyond imagination, and the controllers will switch frequently, which makes it hard to ensure the stability of the closed-loop system, so it is necessary to use the envelope oriented controller design method for RLVs.

\* Resrach supported by Aviation Science Foundation of China under grant #20120151003.

He Chaofan is with the Science and Technology on Aircraft Control Laboratory, Beihang University, Beijing, 100191 China (corresponding author to provide phone: 86-010-8231-6873; e-mail: [hcf.89@163.com](mailto:hcf.89@163.com)).

Yang Lingyu is with the Science and Technology on Aircraft Control Laboratory, Beihang University, Beijing, 100191 China (e-mail: [yanglingyu@buaa.edu.cn](mailto:yanglingyu@buaa.edu.cn)).

Wang Zhenchao is with the Science and Technology on Aircraft Control Laboratory, Beihang University, Beijing, 100191 China (e-mail: [wzc158@163.com](mailto:wzc158@163.com)).

Sun Bin is with the Science and Technology on Aircraft Control Laboratory, Beihang University, Beijing, 100191 China (e-mail: [895367996@qq.com](mailto:895367996@qq.com)).

Zhang Jing is with the Science and Technology on Aircraft Control Laboratory, Beihang University, Beijing, 100191 China (e-mail: [jjizhj1982@163.com](mailto:jjizhj1982@163.com)).

To solve the RLV reentry problems described above, many modern methods are used so far, such as sliding mode control method<sup>[1]</sup>, neural networks direct adaptive control method<sup>[2]</sup>, dynamic inversion control method<sup>[3]</sup> and so on. By contrast, robust LPV control method<sup>[4-6]</sup> is more suitable for RLV attitude controller design for its face-to-envelope design strategy, self-scheduled ability and clear physical significance.

This paper uses the LPV method for a RLV reentry controller design. However, according to the LPV control design principle, the order of LPV controller is related to the LPV vertex model dimension and the number of controller input and output, and the number of LPV vertex controllers is related to the number of LPV vertex model. So the data volume of LPV controller will grow sharply with the increase of dimension and number of LPV vertex model, which makes it difficult to call controller data online and applied to practical engineering.

To decrease the data volume, this paper adopts the relative gain array (RGA) method to analyze the coupling property of a RLV first. A separate design strategy is taken to design the longitudinal controller and lateral controller separately, helping reducing the order of LPV controller. Then by analyzing the eigenvalues of linear model of the RLV, the matrix elements which have a notable impact on the eigenvalues are picked to construct the LPV model, which can reduce the number of LPV vertex model. At last six-degree nonlinear simulation is performed to verify the controller's performance.

## II. RLV SIX DEGREES OF FREEDOM NONLINEAR MODELING

This paper's object of study is Rockwell Space Shuttle numbered SD72-SH-0060. The following assumptions are made.

- The earth is round and considering the earth's rotation, without considering the rotation of the earth around the sun;
- The atmosphere is stationary relative to the earth;
- Aircraft is rigid-body.

Dynamics and kinematics model equations are as follows. Force equations are shown as equation (1).

$$\begin{cases} \dot{V} = (F_x + Se_x + Sc_x) / V \\ \dot{\alpha} = \left[ -(p \cos \alpha + r \sin \alpha) \sin \beta + q \cos \beta - \frac{F_y + Se_y + Sc_y}{mV} \right] / \cos \beta \\ \dot{\beta} = p \sin \alpha - r \cos \alpha + \frac{F_z + Se_z + Sc_z}{mV} \end{cases} \quad (1)$$

In the formulas above,  $F_x, F_y, F_z$  express thrust, gravity and aerodynamic component in the respective airflow coordinate axes;  $Se_x, Se_y, Se_z$  express the inertial force implicated component in the respective airflow coordinate axes;  $Sc_x, Sc_y, Sc_z$  express the Coriolis force of inertia component in the respective airflow coordinate axes .

Moment equations are shown as equation (2).

$$\begin{cases} \dot{p} = \frac{I_z L + I_{xz} N + I_{xz} (I_x + I_z - I_y) pq - (I_z^2 + I_{xz}^2 - I_y I_z) qr}{I_x I_z - I_{xz}^2} \\ \dot{q} = \frac{1}{I_y} [M + (I_z - I_x) pr - I_{xz} (p^2 - r^2)] \\ \dot{r} = \frac{I_x N + I_{xz} L + I_{xz} (I_y - I_x - I_z) qr - (I_x I_y - I_x^2 - I_{xz}^2) pq}{I_x I_z - I_{xz}^2} \end{cases} \quad (2)$$

Motion equations are shown as equation (3).

$$\begin{cases} \dot{\phi} = p + \tan \theta (r \cos \phi + q \sin \phi) \\ \dot{\psi} = (r \cos \phi + q \sin \phi) / \cos \theta \\ \dot{\theta} = -r \sin \phi + q \cos \phi \end{cases} \quad (3)$$

Navigation equations are shown as equation (4).

$$\begin{cases} d\hat{r} / dt = -w \\ d\phi / dt = -v / \hat{r} \\ d\lambda / dt = u / \hat{r} \cos \phi \end{cases} \quad (4)$$

In the formulas above,  $w, u, v$  express speed component in the respective airflow coordinate axes;  $\lambda, \phi, \hat{r}$  respectively express the longitude, latitude and the distance between aircraft and the earth's core.

### III. SELF-SCHEDULED ROBUST LPV CONTROL METHOD

#### A. LPV modeling

A LPV system is called ‘‘polytopic’’ when it can be represented by state-space matrices  $A(\theta(t)), B(\theta(t)), C(\theta(t)), D(\theta(t))$  depending affinely on  $\rho(t)$ , while  $\rho(t)$  varies in a polytope  $\Theta$ . Matrix decomposition is as equation (5).

$$\begin{pmatrix} A(\theta(t)) & B(\theta(t)) \\ C(\theta(t)) & D(\theta(t)) \end{pmatrix} := \sum_{i=1}^r \alpha_i(t) \begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix} \quad (5)$$

where  $A_i, B_i, C_i, D_i$  are state-space matrices of the  $i$ th polytopic LPV vertex models and  $\sum_{i=1}^r \alpha_i = 1$ ,  $A_i$  is a  $m \times m$  matrix,  $B_i$  is a  $m \times n$  matrix,  $C_i$  is a  $p \times m$  matrix,  $D_i$  is a  $p \times n$  matrix.

#### Definition 1 (Polytope system)<sup>[8]</sup>

Polytope system is defined as time varying parameter vector  $\rho(t) \in \mathbb{R}^l$ ,  $l$  is the dimension of  $\rho(t)$  and  $\rho(t) = p(\theta(t))$ , with  $p: \mathbf{R}^k \rightarrow \mathbf{R}^l$  is a mapping function. So the polytope described by the variable parameters is that

$$\Theta := Co\{\omega_1, \omega_2, \dots, \omega_r\} := \left\{ \sum_{i=1}^r \alpha_i \omega_i : \alpha_i \geq 0, \sum_{i=1}^r \alpha_i = 1 \right\} \quad (7)$$

where  $r = 2^l$  is the number of vertex and  $\omega_i$  is the vertex of the polytope.

This paper established aircraft LPV model by using Jacobian linearization. The idea of the method is to obtain LPV system model based on a series of linear model. It is partial approaching to the nonlinear object in a series of equilibrium points. These equilibrium points represent the flight envelope to be studied.

#### B. Self-scheduled strategy

LPV controller can be synthesized by each single vertex controller. The synthesise is based on the location of variable parameter in the polytope.

LPV controller matrix  $A_k(\theta(t)), B_k(\theta(t)), C_k(\theta(t)), D_k(\theta(t))$  is expressed as the following form.

$$\begin{pmatrix} A_k(\theta(t)) & B_k(\theta(t)) \\ C_k(\theta(t)) & D_k(\theta(t)) \end{pmatrix} := \sum_{i=1}^r \alpha_i(t) \begin{pmatrix} A_{ki} & B_{ki} \\ C_{ki} & D_{ki} \end{pmatrix} \quad (10)$$

where  $A_{ki}, B_{ki}, C_{ki}, D_{ki}$  are the  $i$ th vertex controller system matrix, And the calculation of  $\alpha_i$  is as follows.

For a given variable parameter vector

$$\rho = (\rho_1, \dots, \rho_l)^T$$

the convex decomposition coefficients are calculated by

$$\vartheta_i = \frac{(\bar{\rho}_i - \rho_i)}{\bar{\rho}_i - \underline{\rho}_i}, i = 1, \dots, l$$

For each vertex polytope  $\omega_i, i = 1, \dots, r$ , the coefficient

$$\alpha_i = \prod_{i=1}^r \tilde{\vartheta}_i$$

where

$$\tilde{\vartheta}_i = \begin{cases} \vartheta_i & \text{if } \rho_i \in \omega_i \\ 1 - \vartheta_i & \text{if } \bar{\rho}_i \in \omega_i \end{cases}$$

To ensure system stability, weighting coefficient  $\alpha_i$  shall correspond exactly with convex decomposition coefficients. The convex decomposition strategy parameter and weighting coefficient  $\alpha_i$  is calculated based on the geometric distance between the system parameters, which makes the controller able to self-schedule to adapt the whole flight envelope.

In a word, the design steps of the self-scheduled LPV controller are as follows:

- Construct LPV model of nonlinear system;
- Polytope vertex modeling by considering the model parameter uncertainty ;
- Decomposing LPV system model into polytope LPV model by using convex decomposition strategy;
- Obtain the polytope vertex controller using  $H_\infty$  control method based on LMIs;

□ Synthesize robust LPV controller by regarding convex decomposition coefficients as the gain parameter adjustment coefficients to achieve self-scheduled ability in designed envelope.

According to the method above to get LPV controller,  $A_{ki}$  of single LPV vertex controller (as in formula (5)) is a  $(m+n \times j+k) \times (m+n \times j+k)$  matrix,  $B_{ki}$  is a  $(m+n \times j+k) \times p$  matrix,  $C_{ki}$  is a  $n \times (m+n \times j+k)$  matrix,  $D_{ki}$  is a  $n \times p$  matrix, where  $i=1,2,\dots,r$ ,  $r=2^l$ ,  $l$  is the number of variables that are used to construct LPV model,  $k$  is the order of the equivalent transfer function of the steering engine,  $j$  is the order of weighing function. Generally,  $D_{ki}$  is a zero matrix. Therefore, the entire number  $Num$  of LPV controller parameters is

$$Sum = [(m+n \times j+k) \times (m+n \times j+k) + (m+n \times j+k) \times p + n \times (m+n \times j+k)] \times 2^l$$

When the dimension of LPV vertex model is higher and large number of variable parameters are chosen to construct the LPV model, the number of parameters of LPV controller will be grow sharply, so it is necessary to study the simplification of the LPV controller.

#### IV. LPV CONTROLLER DESIGN FOR RLV REENTRY PHASE

The paper designs three channels attitude controllers during RLV reentry region of  $H:30 \sim 40Km$ ,  $Ma:3 \sim 5$ ,  $\alpha:15 \sim 30^\circ$ . From the introduction above, LPV controller is composed of a series of vertices  $H_\infty$  controllers, which are related to the LPV vertex model. LPV model has characteristics with high dimensions and too many vertexes because of the perturbation in a wide range of flight parameter (such as height, mach), and the strong uncertainty of aerodynamic and atmospheric parameter, which will cause the increase of controller order and the number of vertexes and produce a huge data volume of LPV controller which will lose the practical application value. Therefore, when designing the LPV controller for RLV, the paper takes the following measures to keep the controller data volume at a reasonable level.

##### A. Controller structure design based on Relative Gain Array

Coupling characteristic has been analyzed based on Relative Gain Array(RGA) method before the design of controller structure. And this paper adopts a separate design strategy for the longitudinal and the lateral controllers based on the results of the analysis<sup>[11,12]</sup>.

##### a) Principles and process of RGA coupling analysis

RGA originated by Bristol is a useful tool to analyze interactions between input and output variables in multiple input-multiple output (MIMO) control systems.

If a control system with  $n$  input variables and  $p$  output variables exists, the relative gain  $\lambda_{ij}$  between an input variable,  $u_j$ , and an output variable,  $y_i$ , is defined to be the dimensionless ratio of two steady-state gains,

$$\lambda_{ij} = \frac{(\partial y_i / \partial u_j)_u}{(\partial y_i / \partial u_j)_y} = \frac{\text{open-loop gain}}{\text{closed-loop gain}}$$

for  $i=1,2,\dots,p$  and  $j=1,2,\dots,n$ . And  $(\partial y_i / \partial u_j)_u$  denotes a partial derivative when the system is open-loop. Similarly,  $(\partial y_i / \partial u_j)_y$  denotes the system is closed-loop except  $y_i - u_j$  channel.

The RGA is defined as equation (11)

$$R_{RGA} = \begin{matrix} & \begin{matrix} u_1 & u_2 & \cdots & u_n \end{matrix} \\ \begin{matrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{matrix} & \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{p1} & \lambda_{p2} & \cdots & \lambda_{pn} \end{bmatrix} \end{matrix} \quad (11)$$

If  $n = p$ , then

$$R_{RGA} = G(0) \otimes (G(0)^{-1})^T,$$

where  $\otimes$  denotes the Hadamard product. And  $G(0)$  is steady value of the transfer function  $G(s)$ . If  $n \neq p$ , then  $R_{RGA}$  is non-square array,  $G(0)^{-1}$  will be replaced by generalized inverse matrix  $G(0)^*$ .

The RGA has some following important properties,

- Any row or column sums to one.
- If  $\lambda_{ij} = 1$ , it implies that there is no interaction from the other control loops to the  $y_i - u_j$  channel.
- If  $\lambda_{ij} = 0$ , it implies that the input variable  $u_j$  has no effect on the output variable  $y_i$ .
- If  $0 < \lambda_{ij} < 1$ , it implies that there is some interaction from the other control loops to the  $y_i - u_j$  channel. When the value of  $\lambda_{ij}$  is getting close to 1, the interaction from other channels weakens.
- If  $\lambda_{ij} < 0$  or  $\lambda_{ij} > 1$ , the system will be unstable and this situation needs to avoid.

The state space matrix is as follows,

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (12)$$

where  $x = [q \ r \ p]^T$ ,  $q, r$  and  $p$  respectively denotes pitch rate, yaw rate and roll rate, and  $u = [\delta_m \ \delta_n \ \delta_l]^T$ ,  $\delta_l, \delta_n$  and  $\delta_m$  respectively denotes the equivalent rudder of roll channel, yaw channel and pitch channel. Based on RGA theory, the RGA is

$$R_{RGA} = \begin{matrix} & \begin{matrix} \delta_m & \delta_n & \delta_l \end{matrix} \\ \begin{matrix} q \\ r \\ p \end{matrix} & \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} \end{matrix}. \quad (13)$$

The paper analyzes the coupling characteristics as the mach number gradually increases, while  $\alpha = 15^\circ$  and  $30^\circ$ ,  $H = 30000m$ ,  $\beta = 0^\circ$ .

### 1) Coupling characteristics analysis with small angle of attack

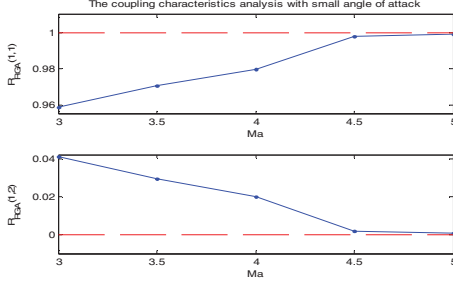


Figure 1. Coupling characteristics analysis with small angle of attack

In Figure 1, the values of  $\lambda_{11}$  and  $\lambda_{22}$ , which are greater than 1 in the beginning, decreases closely to 1, while the values of  $\lambda_{12}$  and  $\lambda_{21}$  close to 0. It indicates that the interaction in the lateral channels gradually weakens as the Mach number gradually increases, while there is no coupling between the longitudinal channel and the lateral channels.

### 2) Coupling characteristics analysis with high angle of attack

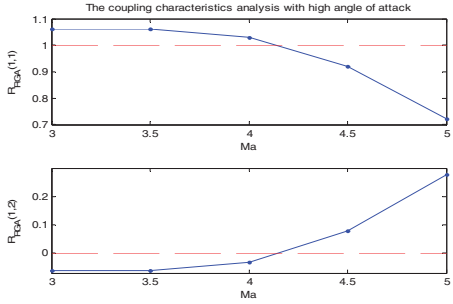


Figure 2. Coupling characteristics analysis with high angle of attack

In Figure 2, the values of  $\lambda_{11}$  and  $\lambda_{22}$ , which are greater than 1 in the beginning, decreases gradually and they are less than 1 at last. The values of  $\lambda_{12}$  and  $\lambda_{21}$  increase and eventually are greater than 0. However the value of  $\lambda_{ii}$  is gradually away from 1, which indicates that the coupling in the lateral channels as the Mach number gradually increases, while there is no coupling between the longitudinal channel and the lateral channels.

### b) Separate design of the longitudinal and lateral controllers

Through coupling analysis, there is no coupling between the longitudinal channel and the lateral channels, but it is serious in the lateral channels. So the paper designs the controllers separately to reduce the controller order and design difficulty.

The RLV longitudinal attitude channel linear model can be expressed as,

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} [\delta_m] \quad (15)$$

The RLV lateral attitude channels linear model can be expressed as,

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{\gamma} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} a_{33} & a_{34} & a_{35} & a_{36} \\ a_{43} & a_{44} & a_{45} & a_{46} \\ a_{53} & a_{54} & a_{55} & a_{56} \\ a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \gamma \\ p \end{bmatrix} + \begin{bmatrix} b_{31} & b_{32} \\ b_{41} & b_{42} \\ b_{51} & b_{52} \\ b_{61} & b_{62} \end{bmatrix} \begin{bmatrix} \delta_l \\ \delta_n \end{bmatrix} \quad (16)$$

The controller structure is shown in Figure 3.

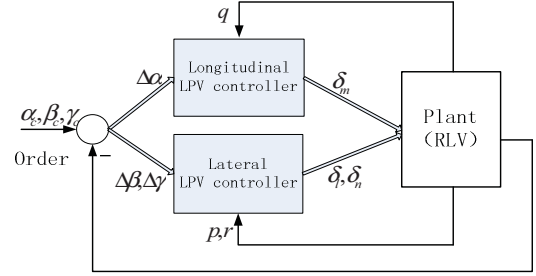


Figure 3. LPV controller structure

### B. Polytope vertex construction based on coupling analysis

As equation (15) and (16) shows, the LPV model of longitudinal channel and lateral channels are second order and fourth order. Choosing the elements of state matrix and control matrix to be the variable parameters to construct the LPV model, is unrealistic because of too many vertices. So firstly, the paper trims the nonlinear model at each state point within the designed envelope to get a series of linear model. Then from the following two aspects of analysis, choose the most significant elements on linear models to be the variable parameters which construct the LPV model. Thus it will greatly reduce the number of vertices in LPV model.

- Analyzing the impact on model eigenvalues of the elements in the state matrix when they change from their upper to their lower bound.
- Analyzing the changes in the amplitude and range of each elements of the control matrix in the designed envelope.

From the analysis, the values of  $a_{45}$ ,  $a_{55}$ ,  $a_{65}$  and  $a_{53}$  are the constant value 0, so these elements can be ignored while constructing the LPV model. And in the state matrix  $a_{21}$ ,  $a_{55}$  and  $a_{63}$  have obvious influence on the model eigenvalues. Meanwhile  $b_{13}$ ,  $b_{23}$  and  $b_{61}$  change within large range.

Therefore  $a_{21}$ ,  $b_{13}$  and  $b_{23}$  are chosen as the variable parameters to construct the LPV model in the longitudinal channel, while  $a_{55}$ ,  $a_{63}$  and  $b_{61}$  are chosen in the lateral channels. The controllers of  $\alpha$  channel and  $\beta\gamma$  channels are expressed as

$$\begin{pmatrix} A_{k_\alpha}(\theta(t)) & B_{k_\alpha}(\theta(t)) \\ C_{k_\alpha}(\theta(t)) & D_{k_\alpha}(\theta(t)) \end{pmatrix} := \sum_{l=1}^p a_{\alpha,l}(\theta(t)) \begin{pmatrix} A_{k_{\alpha,l}} & B_{k_{\alpha,l}} \\ C_{k_{\alpha,l}} & D_{k_{\alpha,l}} \end{pmatrix}$$

$$\begin{pmatrix} A_{k_{\beta\gamma}}(\theta(t)) & B_{k_{\beta\gamma}}(\theta(t)) \\ C_{k_{\beta\gamma}}(\theta(t)) & D_{k_{\beta\gamma}}(\theta(t)) \end{pmatrix} := \sum_{l=1}^q a_{\beta\gamma,l}(t) \begin{pmatrix} A_{k_{\beta\gamma,l}} & B_{k_{\beta\gamma,l}} \\ C_{k_{\beta\gamma,l}} & D_{k_{\beta\gamma,l}} \end{pmatrix}$$

where  $\begin{pmatrix} A_{k_{\alpha,l}} & B_{k_{\alpha,l}} \\ C_{k_{\alpha,l}} & D_{k_{\alpha,l}} \end{pmatrix}$  and  $\begin{pmatrix} A_{k_{\beta\gamma,l}} & B_{k_{\beta\gamma,l}} \\ C_{k_{\beta\gamma,l}} & D_{k_{\beta\gamma,l}} \end{pmatrix}$  are the controllers of given vertexes. Meanwhile  $p = 2^3, q = 2^3$ .

At last, the comparison of data volume between the standard LPV controller and the simplified LPV controller, is as shown in Table 1.

TABLE I. THE COMPARISON OF DATA VOLUME

	Simplification		No Simplification
	Longitudinal	Lateral	
LPV vertex model order	2	4	6
Controller order	4	8	12
No. of variable parameters	3	3	24
No. of LPV vertex model	8	8	224
Data volume of LPV model	1120		$4 \times 10^9$

Through the comparison, after channel-separate-design and reduction in the number of model vertexes, the total amount of stored LPV model data significantly reduces and makes the LPV controller more applicable.

## V. SIMULATION

### A. Simulation under some fixed flight state point

The flight state is  $H=35\text{Km}$ ,  $Ma=4$ ,  $\alpha = 15^\circ$ ,  $\beta = 0^\circ$ . The simulation results under nominal situation are shown as the Figure 4~ 6. We can see the simplified LPV controller can track the angle of attack, sideslip and roll command precisely without considering any uncertain condition.

Considering the uncertainty of aerodynamic force and moment coefficients, as well as the uncertainty of mass and rotational inertia, as shown in Table 2, 100 Monte-Carlo simulations are run on the closed loop system. The results are shown as Figure 7, indicating the strong robustness of the simplified LPV controller.

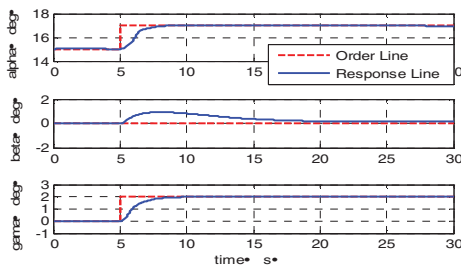


Figure 4. Three channels' step response curves

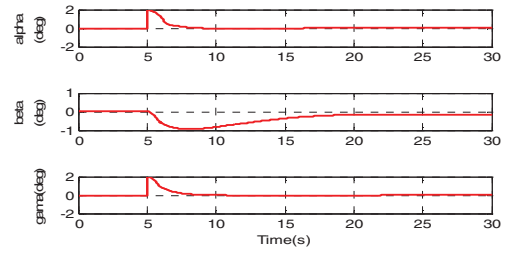


Figure 5. Three channels' tracking error curves

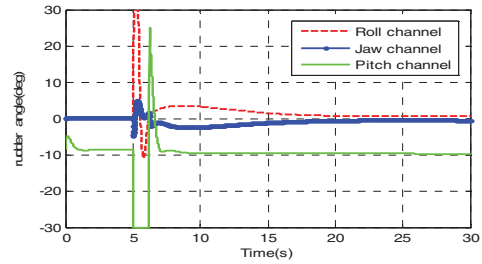


Figure 6. Three channels' equivalent rudder bias curves

TABLE II. UNCERTAINTY

Deviation Item	Range of Uncertainty
Aerodynamic force coefficients	-20%~+20%
Aerodynamic moment coefficients	-50%~+50%
Mass	-3%~+3%
Rotational inertia	-5%~+5%

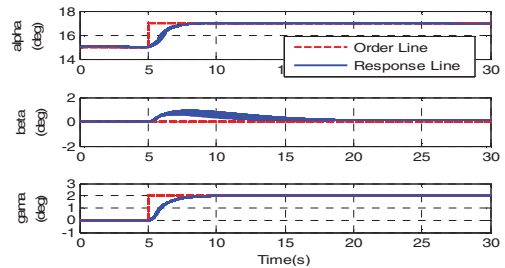


Figure 7. 100 Monte-Carlo simulations result

### B. Simulation within wide envelope

According to the flight state diagram in Reference [7], the flight commands are extracted, and used for the closed loop simulation. The flight envelope is shown an Figure 8, the simulation results are shown as Figure 9-10. We can see the simplified LPV controller can track the three channels' command precisely within the whole envelope, which indicates the controller's adaptive ability.

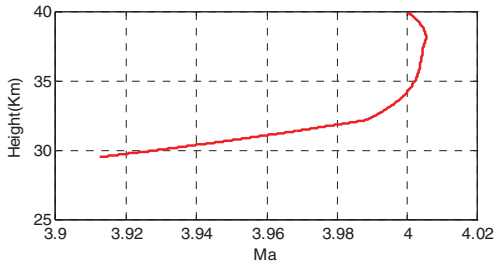


Figure 8. Flight envelope

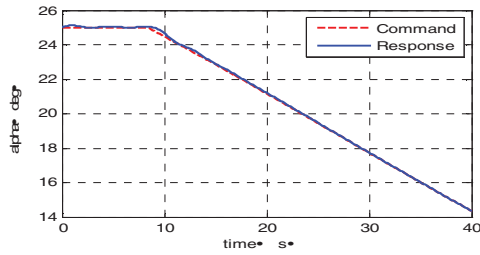


Figure 9. Longitudinal channel's simulation result

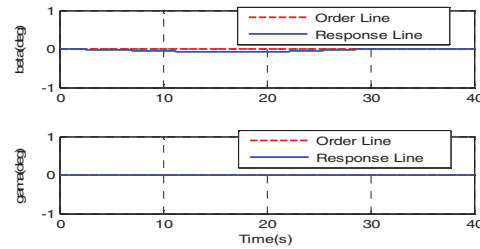


Figure 10. Lateral channels' simulation result

100 Monte-Carlo simulations are run on the closed loop system considering the uncertainty shown as Table 2. The results are shown as Figure 11, which indicates the strong robustness of the simplified LPV controller.

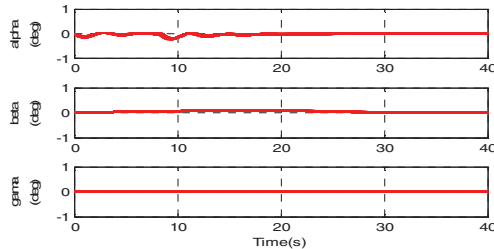


Figure 11. Tracking error curves of Monte-Carlo simulations

## VI. CONCLUSION

Taking the strong uncertainty and coupling of the RLV during reentry into consideration, this paper chooses LPV method to design the attitude controller. To improve the controller's engineering practicability and reduce the order of LPV controller, the degree of coupling between channels is analyzed first, and the result contributes to the development of scheme of LPV controller, which is the separate design of longitudinal controller and lateral controller. Besides, this paper chooses several key parameter of the linear model to

construct the LPV model, by analyzing their influence degree to the model's characteristics, which also contributes to the decrease of data volume if LPV controller. By comparison the simplified with the standard LPV controller's data volume, we can see the data volume of the simplified is far smaller. And through the simulation of 6-degree nonlinear closed system, we can see the robustness and tracking performance of the simplified LPV controller are guaranteed, which indicates the rationality of the simplification methods.

## ACKNOWLEDGMENT

This work is supported by Aviation Science Foundation of China (Grant No. 20120151003).

## REFERENCES

- [1] C.E.Hall, Y.B Shtessel, "Sliding mode disturbance observer-based control for a reusable launch vehicle," *Journal of guidance, control, and dynamics*, Vol.29, No.6, November–December 2006
- [2] EN Johnson, AJ Calise, JE Corban, "Reusable launch vehicle adaptive guidance and control using neural networks," *AIAA Guidance, Navigation and Control Conference and Exhibit*, AIAA 2001-4381.
- [3] Bai Chen, Ren Zhang, FAN Yao. "Dynamic inversion control for RLV reentry attitude based on fuzzy-neural disturbance observer," *Journal of Central South University*, 2013,7(1), pp: 58-62.
- [4] Lind R, "Linear parameter-varying modeling and control of structural dynamics with aerothermoelastic effects. *Journal of Guidance, Control and Dynamics*, 2002, 25(4):733-739.
- [5] P.P.Menon, E.Prempan, I.Postlethwaite, D.Bates, "Nonlinear Worst-Case Analysis of an LPV Controller for Approach-Phase of a Re-Entry Vehicle," *AIAA GNC*, 2009
- [6] Nicolas Fezans, "Robust LPV control design for a RLV during reentry," *Scientist*, 2010, 5(25.27): 86.
- [7] Aerodynamic design data book. Volume 1M: Orbiter vehicle STS-1,SD72-SH-0060, *Rockwell International*, 1980
- [8] Apkarian P, Gahinet P, Becker G, "Self-scheduled  $H_\infty$  Control for a linear parameter varying system: a Design Example," *Automatic*, 1995,31(9):11-21.
- [9] Gahinet P, Nemirovski A, Laub Alan J. et al, LMI Control Tool box, 1sted. *Massachusetts: The Math Works, Inc*, 1995:7.2-7.15.
- [10] Li Wenqiang, Zheng Zhiqiang, "Robust Gain-Scheduling Controller to LPV system Using Gap Metric," *Proceedings of the 2008 IEEE International Conference on Information and Automation, International Conference on. IEEE*, 2008: 514-518.
- [11] Witcher M, Mcavoy T J. "Interacting control systems: steady state and dynamic measurement of interaction," *ISA Trans*, 1977, 16:83-90.
- [12] Bristol, E. "On a new measure of interaction for multivariable process control," *IEEE Transactions on Automatic Control*, 1966, 11(1): 133-134.
- [13] Marcos, A. Balas, G.J., "Development of Linear-Parameter-Varying Models for Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol.27, No.2, 2004, pp.218–228.
- [14] Apkarian P, Gahinet P, "A convex characterization of gain scheduled  $H_\infty$  controllers," *IEEE Transactions on Automatic Control*, 1995,40(5):853-864.