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Xiaoke Yang, Mingqiang Luo, Jing Zhang, Lingyu Yang

School of Aeronautic Science and Engineering, Beihang University

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Aircraft Centre-of-Gravity Estimation using Gaussian Process Regression Models

Xiaoke Yang*, Mingqiang Luo*, Jing Zhang*, and Lingyu Yang

***Abstract*— Aircraft centre of gravity (C.G.) is important for aircraft safety and performance. This paper proposes the use of Gaussian process regression (GPR) models for the estimation of the C.G. location of fixed-wing aircraft. The major benefit of using a GPR model is that it is a data-based approach explicitly tackling uncertainties caused by the quality and quantity of the data as well as sensor measurement noise. The proposed method consists of two steps: the estimation of the fuel tank's C.G. using the GPR model trained with fuel weight property data, and the computation of aircraft C.G. by the C.G. equation. A numerical case study of a transport aircraft shows that the proposed method achieves small mean squared error and gives good estimate of the aircraft C.G. under simulated flight scenarios.**

***Index Terms*— Centre-of-Gravity Estimation, Gaussian Process, Kriging Interpolation, Aircraft Fuel System, Aircraft Weight and Balance**

I. Introduction

The location of the centre of gravity (C.G.) is crucial for the stability and performance of an aircraft. Fuel consumption, fuel transfer, movement or jettison of the payload etc. could all affect the C.G. location, posing challenges to the quality and safety of the flight. While the location of the C.G. cannot be accurately measured, it is thus necessary to estimate it online, in order to provide information for the flight control system and flight monitoring procedures.

Conventional approaches to the estimation of the aircraft C.G. utilises the weight and balance (W&B) data from different component groups, e.g. the fuselage group, the wing group, etc.[1]. The C.G. of the aircraft is then determined by these component groups through the formula of the C.G. for a collection of point masses. This method is widely used in industry for different types of aircraft due to its simplicity and generality. But the accuracy of the method is restricted by potential noise and errors in the measurement and computation of the W&B data. And this procedure is not flexible enough for an in-flight reset to cope with contingencies such as erroneous measurement of critical sensors [2], [3].

To overcome the shortcomings of the conventional method, various in-flight estimation schemes which do not rely on the offline W&B data are proposed in literature. [3] and [4] patented an estimation method of the longitudinal location of the C.G. for aircraft with adjustable horizontal stabilisers. The method uses the aircraft structural data, e.g. location of the focus, the flight data available, e.g. Mach number, engine speed, and lift coefficient, and the measurement of the deflection of the horizontal stabiliser to determine the longitudinal location of the C.G. This complicated nonlinear mapping is decomposed into simpler functions, which are constructed by experimental data and theoretical analysis. [5] proposed to use artificial neural networks to estimate aircraft longitudinal C.G. locations in trimmed symmetric flight conditions. The neural network takes the Mach number, the angle of attack, the flight path angle, and the elevator deflections as the input and issues the longitudinal C.G. location and weight as the output. Training data for the neural network come from simulated flight tests under 'a set of weights, longitudinal C.G. positions, Mach numbers, altitudes, and throttle settings'[5]. These two methods do not rely on aircraft W&B data, and the estimation of the C.G. location is computed solely from the nonlinear mappings obtained beforehand. The accuracy is improved and the computation can be easily restarted in flight. However, the use of these methods is mainly restricted to cruise or trimmed flight due to their underlying principles, and the methods are also not mature enough for industrial implementation.

Recently, [6] proposed a combination of the conventional method and the neural network method in [5]. Estimates from both methods are passed through a data sorting algorithm, which selects the appropriate data source given the flight conditions. The selected estimates are then fed into an adaptive weighted fusion algorithm to compute the final C.G. estimate. Both the estimation accuracy and the robustness against noisy and erroneous data are improved.

In contrast to the methods in [3], [5], and [6], this paper follows the conventional W&B data-based track and proposes the use of Gaussian process regression (GPR) models, or Kriging interpolation as called in Geostatistics [7], to estimate the aircraft C.G. location. Facing the disadvantages of the conventional approach, e.g. the noise and errors in the data and sensor measurement, the proposed method does not aim to overcome this restriction, but rather tries to incorporate it as uncertainties and give a probabilistic estimate of the C.G. Furthermore, with the aid of advanced computing and manufacturing techniques, such as computer-aided design and computer-aided manufacturing, an accurate and detailed W&B database of aircraft component groups,

*These authors contributed equally.

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Xiaoke Yang, Jing Zhang, and Lingyu Yang are with the School of Automation Science and Electrical Engineering, Beihang University, Beijing, 100191, China. email: das.xiaoke@hotmail.com, jijizhj1982@163.com, yanglingyu@buaa.edu.cn.

Mingqiang Luo is with the School of Aeronautic Science and Engineering, Beihang University, Beijing, 100191, China. email: luomingqiang@buaa.edu.cn.

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especially the fuel system, has become more and more viable [8]. The main contribution of the proposed method is the application of a data-based probabilistic modelling method for the estimation of aircraft C.G. location. Within this method, an approximation is used to improve its real-time performance. Errors and noise in the data and measurement are also considered.

The rest of the paper is organised as follows. Section II summaries the necessary background on Gaussian processes. Section III then discusses the proposed method on aircraft centre-of-gravity estimation in detail. Section IV gives a case study and presents the results. Conclusions and further work then follow in Section V.

II. Background

A. Gaussian Process Regression Models

A nonlinear regression problem can be stated as identifying the input-to-output mapping $f(\mathbf{x})$ of a system

$$y = f(\mathbf{x}) + \epsilon \tag{1}$$

given input-output data $\mathcal{D} = \{\mathbf{X}, \mathbf{y}\}$, ϵ is a noise term. A Gaussian process (GP) provides a probabilistic way of describing such a mapping by defining a distribution over functions[9]. Similar to a multivariate Gaussian distribution characterised by a mean vector and a covariance matrix, a Gaussian process is fully specified by a mean function $m(\cdot)$ and a covariance function $k(\cdot,\cdot)$, and is denoted as

$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}')), \tag{2}$$

where $\mathbf{x}$ and $\mathbf{x}'$ are two input points. The mean function specifies the 'average shape' of the function. The covariance function specifies the covariance between any two function values, computed from the corresponding inputs.

Given input-output data $\mathcal{D} = \{\mathbf{X}, \mathbf{y}\}$, the identified function, or the posterior GP is written as

$$f(\mathbf{x}) \sim \mathcal{GP}(m_+(\mathbf{x}), k_+(\mathbf{x}, \mathbf{x}')), \tag{3}$$

where

$$m_+(\mathbf{x}) = m(\mathbf{x}) + k(\mathbf{x}, \mathbf{X}) \left[k(\mathbf{X}, \mathbf{X}) + \sigma_\epsilon^2 \mathbf{I}\right]^{-1} (\mathbf{y} - m(\mathbf{X})), \tag{4a}$$

$$k_+(\mathbf{x}, \mathbf{x}') = k(\mathbf{x}, \mathbf{x}') - k(\mathbf{x}, \mathbf{X}) \left[k(\mathbf{X}, \mathbf{X}) + \sigma_\epsilon^2 \mathbf{I}\right]^{-1} k(\mathbf{X}, \mathbf{x}'). \tag{4b}$$

Then for an input $\mathbf{x}_*$, the corresponding function value has a Gaussian distribution as

$$f(\mathbf{x}_*) \sim \mathcal{N}(m_+(\mathbf{x}^*), k_+(\mathbf{x}_*, \mathbf{x}_*)). \tag{5}$$

Parameters of the mean function and the covariance function $\boldsymbol{\theta}$ are called the hyper-parameters of the GP. An 'optimal' value of the hyper-parameters under data set $\mathcal{D}$ is usually computed by minimising the following negative logarithm marginal likelihood,

$$\hat{\boldsymbol{\theta}} \in \arg\min_{\boldsymbol{\theta}} \frac{1}{2}\mathbf{y}^{\mathrm{T}}\mathbf{K}_{\boldsymbol{\theta}}^{-1}\mathbf{y} + \frac{1}{2}\log|\mathbf{K}_{\boldsymbol{\theta}}| + \frac{1}{2}D\log(2\pi), \tag{6}$$

where $\mathbf{K}_{\boldsymbol{\theta}} = k(\mathbf{X}, \mathbf{X}) + \sigma_\epsilon^2\mathbf{I}$. This procedure is referred to as the training of the GP.

B. Computational Complexity Reduction of the GP regression model

For a GP regression model with N pairs of input-output training data, the computational complexity of the training is $\mathcal{O}(N^3)$, dominated by the matrix inversion in (6). The computation of $f(\mathbf{x}_*)$ in (5) requires $\mathcal{O}(N^2)$ operations, mainly on the matrix products in (4a) and (4b). Since the training of a GP regression model is mostly performed offline, the $\mathcal{O}(N^3)$ complexity will not pose difficulties for online predictions. However, as the size of the training data increases, even the $\mathcal{O}(N^2)$ complexity will slow down the online prediction.

Fully Independent Training Conditional (FITC) approximation to GP in [10] aims to reduce the online prediction complexity by introducing a much smaller set of M ($M < N$) pseudo 'data points', i.e. pseudo inputs $\bar{\mathbf{X}} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_M\}$ with associated pseudo function values $\bar{\mathbf{f}}$. The predicted function output can be computed by the pseudo inputs $\bar{\mathbf{X}}$ and precomputed covariance matrices between the data and the pseudo points at a reduced cost of $\mathcal{O}(M^2)$. Determination of the locations of the pseudo-inputs $\bar{\mathbf{X}}$ can be done offline by maximising the likelihood $p(\mathbf{y}|\mathbf{X}, \bar{\mathbf{X}}, \boldsymbol{\theta})$.

C. GP Regression Models with Input Noise

In (5), the test input $\mathbf{x}_*$ has a deterministic value. There are also cases where the test input is uncertain. For example, when $\mathbf{x}_*$ comes from a Bayesian filter or a probabilistic fault identification procedure, a distribution (assumed to be Gaussian) $\mathbf{x}_* \sim \mathcal{N}(\boldsymbol{\mu}_{\mathbf{x}_*}, \boldsymbol{\Sigma}_{\mathbf{x}_*})$ will appear over $\mathbf{x}_*$. In this case, marginalisation over $\mathbf{x}_*$ should be carried out to get the distribution of the function output [11], i.e. $p(f_*) = \int p(f(\mathbf{x}_*)|\mathbf{x}_*)p(\mathbf{x}_*)\mathrm{d}\mathbf{x}_*$. This marginalisation will not lead to a Gaussian distribution in general, and Gaussian approximation through exact moment matching is usually performed as

$$\mu_{f_*} = \mathbb{E}_{\mathbf{x}_*}\left[m_+(\mathbf{x}_*)\right], \tag{7}$$

$$\sigma_{f_*}^2 = \mathbb{E}_{\mathbf{x}_*}\left[k_+(\mathbf{x}_*, \mathbf{x}_*) + m_+(\mathbf{x}_*)^2\right] - \mu_{f_*}^2. \tag{8}$$

III. Aircraft Centre-of-Gravity Estimation using Gaussian Process Regression Models

A. The Aircraft Centre-of-Gravity Estimation Scheme

Weight and balance data for each component group of an aircraft, including the weight and the centre of gravity, are recorded and updated during the design and manufacturing stage. This W&B data are then used to compute the overall weight and C.G. of the aircraft. Within all the component groups, the weight and C.G. of the aircraft's structural group, power plant, and systems group rarely change in flight. While the payload, i.e. the passengers and the cargo, and the fuel are the two factors causing the C.G. to move in flight, and the fuel consumption is the dominating factor.

The proposed estimation scheme of the C.G. of an aircraft is thus decomposed into two steps: the estimation the C.G. of the fuel tanks and the computation of the aircraft's C.G. by the results of the first step and the aircraft zero-fuel weight and zero-fuel C.G.. A block diagram of this scheme is in Fig.

1. For the first step, a GP regression model is trained with the weight property database of the fuel tanks during the offline training stage. After training, the hyper-parameters and the pseudo points are used in the GPR model for the online estimation of the fuel C.G. The second step follows the conventional approach by using the C.G. equation for the aircraft C.G. location.

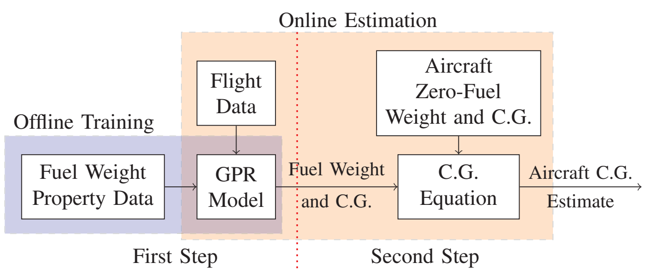

Fig. 1: Block diagram of the proposed aircraft C.G. estimation method using GP regression models.

B. Estimating the Centre of Gravity of the Fuel Tanks

1) Weight property database of the fuel tanks: Since fuel consumption is the major factor causing the weight and C.G. of the aircraft to change in flight, the analysis of the weight property of the fuel tanks is essential and is iterated during the whole span of aircraft design and manufacturing. Analytical approaches, such as empirical formulae or approximating the fuel tanks by simple curves, surfaces, and volumes, have been used in the early years. While with the progress of computing techniques, especially with the assistance of fuel tank structural geometry CAD database and finite element analysis method, refined numerical integration is performed by slicing the fuel tanks with infinitesimal steps, generating much more accurate weight property data.

For a specific fuel tank, the weight property database includes entries of fuel weight, C.G., and moment of inertia under arbitrary fuel quantity and fuel surface angle. The fuel surface angle is further affected by the attitude and the acceleration of the aircraft. Each entry is computed under a specific combination of aircraft attitude, acceleration, and fuel quantity. The span of the combination needs to cover all possible values of the flight parameters and the fuel quantity.

2) GP regression model for the fuel tanks database: The fuel weight property database describes a nonlinear mapping for each fuel tank as

$$\mathbf{y} = \mathbf{f}(\mathbf{x}), \tag{9}$$

where the input $\mathbf{x} \in \mathbb{R}^D$ is a column vector, containing the attitude and the three-axis acceleration of the aircraft as well as the fuel quantity in the tank. The output variable $\mathbf{y} \in \mathbb{R}^E$ consists the three-axis C.G. of the fuel tank with respect to a certain reference point. As described in section II, a Gaussian process model could be used to describe this nonlinear mapping. i.e.

$$f_i \in \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}')), i \in \{1, \ldots, E\}, \tag{10}$$

where $m(\mathbf{x}) = c$ is chosen as a constant function, in which the constant c is the hyper-parameter. $k(\mathbf{x}, \mathbf{x}')$ is chosen to be the squared-exponential covariance function

$$k(\mathbf{x}, \mathbf{x}') = \sigma_f^2 \exp\left(-\frac{1}{2}(\mathbf{x} - \mathbf{x}')^\top \mathbf{\Lambda}^{-1}(\mathbf{x} - \mathbf{x}')\right), \tag{11}$$

where $\mathbf{\Lambda}$ is a diagonal matrix. The diagonal elements of Λ and σ_f are the hyper-parameters. The combination of a constant mean function and (11) makes the GPR model a universal approximator to smooth nonlinear functions.

Since the weight property data from numerical computation do not have measurement noise, there is no need for a noise term in (9). In case there are data from sensor measurement, a noise term could be appended to (9) and a Gaussian likelihood function can be used accordingly, with the variance term as an additional hyper-parameter.

Then the GPR model in (10) can be trained by the weight property data as through (6), in order to find an optimal value for the hyper-parameters.

3) Reducing the computational load: The weight property database usually contains tens of thousands of records, and online prediction using a GPR model over such a database involves significant computational load. FITC approximation technique is used in the proposed scheme, in order to meet the high real-time demands of C.G. estimation. Using FITC approximation means that in the offline training stage, after the hyper-parameters are optimised, the pseudo inputs are also computed and passed to the GPR model for online estimation.

4) Interfacing with noisy and erroneous input: For the estimation of the fuel tanks' C.G. location, input variables to the nonlinear function $\mathbf{f}(\mathbf{x})$ include aircraft attitude, acceleration, and the fuel quantity, whose measurement may suffer from noise or errors. A Bayesian filter or a fault identification filter could be devised in such situations to provide a probabilistic reconstruction of the faulty or noisy variables, given measurement of other variables and the flight dynamics of the aircraft. The proposed C.G. estimate method does not include such a filter but provides an interface for it through input distributions. This interface accounts the mean of the input variable as well as the variance, which is an indication of the quality of the input. The resulting C.G. estimate is certainly not perfect, but a more informative estimate is provided. Incorporating the variance of the input variables requires the techniques in section II.C, and the computation is carried out online.

C. Estimating the Centre of Gravity of the Aircraft

After obtaining the C.G. estimate and the weight of the fuel tanks, computing the C.G. of the aircraft is straightforward. By defining a common reference point for all the fuel tanks and the zero-fuel aircraft, the C.G. of the aircraft is computed as

$$\mathbf{x}_a = \frac{\sum_{i=1}^n m_i \mathbf{x}_i + m_{\mathrm{ZF}} \mathbf{x}_{\mathrm{ZF}}}{\sum_{i=1}^n m_i + m_{\mathrm{ZF}}}, \tag{12}$$

where n is the number of fuel tanks, $\mathbf{x}_i$ and m_i are the C.G and weight of the i^{th} fuel tank respectively. $\mathbf{x}_i \sim$

$\mathcal{N}(\boldsymbol{\mu}_{\mathbf{x}_i}, \boldsymbol{\Sigma}_{\mathbf{x}_i})$ is a vector of Gaussian random variables. m_{ZF} and $\mathbf{x}_{\text{ZF}}$ are the weight and the C.G. of the zero-fuel aircraft, respectively.

Since linear combinations of independent Gaussian random variables is also a Gaussian random variable, the distribution of the aircraft C.G. location is $\mathbf{x}_a \sim \mathcal{N}(\boldsymbol{\mu}_{\mathbf{x}_a}, \boldsymbol{\Sigma}_{\mathbf{x}_a})$, where

$$\boldsymbol{\mu}_{\mathbf{x}_a} = \frac{\sum_{i=1}^{n} m_i \boldsymbol{\mu}_{\mathbf{x}_i} + m_{\text{ZF}} \mathbf{x}_{\text{ZF}}}{\sum_{i=1}^{n} m_i + m_{\text{ZF}}}, \tag{13}$$

$$\boldsymbol{\Sigma}_{\mathbf{x}_a} = \frac{\sum_{i=1}^{n} m_i^2 \boldsymbol{\Sigma}_{\mathbf{x}_i}}{\left(\sum_{i=1}^{n} m_i + m_{\text{ZF}}\right)^2}. \tag{14}$$

IV. Case Study: Centre-of-Gravity Estimation of a Transport Aircraft

In this section, a case study of the proposed C.G. estimation method is carried out on a transportation aircraft. The layout of the aircraft's fuel tanks is in Fig. 2.

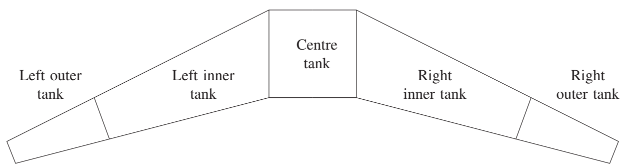

Fig. 2: Layout of the fuel tanks.

There are five tanks in total: one centre tank within the fuselage, and four wing tanks. The fuel weight property data has 5 tables, each containing 10,759 rows of scattered data points over irregular grids. Fig. 3 shows how the 10,759 data points of the centre tank are scattered.

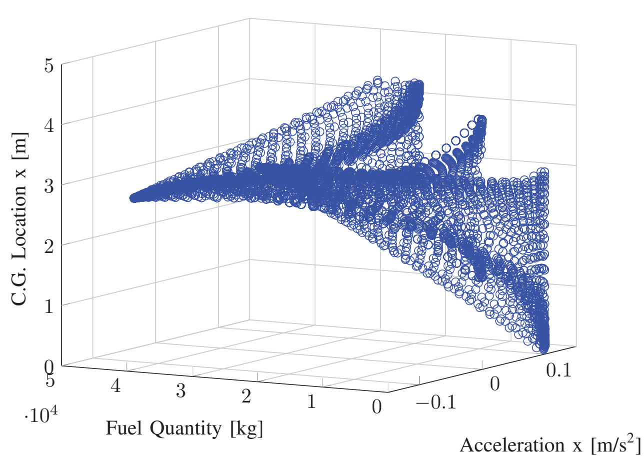

Fig. 3: Scatter plot the the data points in the fuel weight property data of the centre tank.

In accordance with the two steps in the proposed method, the case study first inspects the estimation of the C.G. of the fuel tanks, then proceeds to the estimation of the aircraft C.G. by flight simulations.

A. Estimating the C.G. of the Fuel Tanks

The data for each fuel tank are split evenly into 80% training data and 20% test data. For each fuel tank, 3 GPR models are trained, corresponding to the three-axis C.G. locations.

The computation was carried out on a laptop with a 2.9 GHz Intel Core i5 processor and 4 GB of memory. For the GP regression model, the GPML toolbox [9] was used. The training of the GP uses a conjugate gradient method with 100 iterations. Fig. 4 shows the negative log-marginal likelihood (NLML) of the GPR models for the longitudinal C.G. of the five fuel tanks when the hyper-parameters are trained.

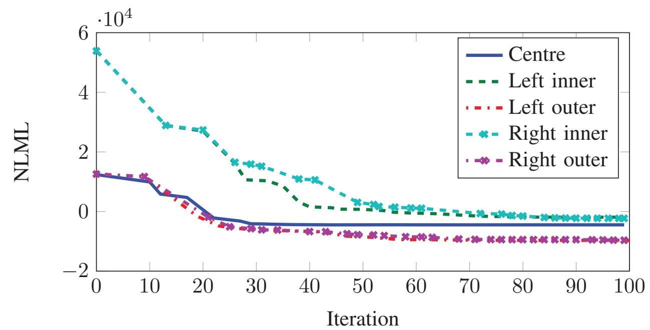

Fig. 4: Negative log-marginal likelihood of the GPR models for the longitudinal C.G. during the offline training operation.

For the FITC approximation, the pseudo points are not computed through optimisation due to a prohibitively large number of decision variables: the number of decision variables is 4 times of the number of pseudo points. Instead, analysis of the data shows relatively dense grid points along the fuel quantity input, thus 5 equally-spaced pseudo points are used to replace the original 24 data points for each combination of the other inputs. This gives 1,851 pseudo points for the GPR model with reasonably small NLML.

Since most curve-fitting and interpolation methods use the mean squared error (MSE) as an index for the fitting performance. The values of NLML do not translate directly into MSE, but an MSE between the predicted mean values of the GPR model and the actual data output values can still be computed. Table I lists the average computational cost and the MSE of the GPR model for the longitudinal C.G. of the left inner tank.

TABLE I: Time consumption and fitting performance of the GPR model for the longitudinal C.G. of the left inner tank.

Item	**Value**
Average time for training the hyper-parameters	7223 s
Average time of GPR prediction without FITC	9.7 s
Average time of GPR prediction with FITC	*3.8 s*
MSE of the training data set without FITC	*5.8e-3*
MSE of the test data set without FITC	*1.4e-2*
MSE of the training data set with FITC	*2.6e-2*
MSE of the test data set with FITC	*3.9e-2*

It can be seen from Table I that the training of the GPR model is the most time-consuming operation, which takes about 5 hours in total. But this 5 hours is spent offline and could be reduced if more powerful computers are used. For the prediction part, the full GPR model takes about 9.7

seconds to give a single estimate. In contrast, with FITC approximation, the time for a single prediction is reduced to less than a half, i.e. about 3.7 seconds. Also, the MSEs of the GPR models on both the training data set and the test data set are at a relatively low level, indicating a good regression performance.

B. Estimating the C.G. of the Aircraft

After the GPR models for the fuel tanks have been trained, the C.G. of the whole aircraft can be computed. Two flight simulations are carried out in this part, including decelerated level flight and cruise flight. The first scenario is to test the C.G. estimation with respect to an acceleration input while the second to a fuel quantity input.

1) Decelerated Level Flight: This flight scenario is initiated by a step input to the throttle lever for 20 seconds corresponding to -0.2 g deceleration. The acceleration input has a Gaussian distribution to account for the Gaussian measurement noise. The estimate of the aircraft C.G. variation from the proposed method is shown in Fig. 5. The shaded band in the figure shows the 95% confidence interval. Since the true values of the C.G. locations are unavailable, results from a 1-D spline interpolation using the local data are used as a reference.

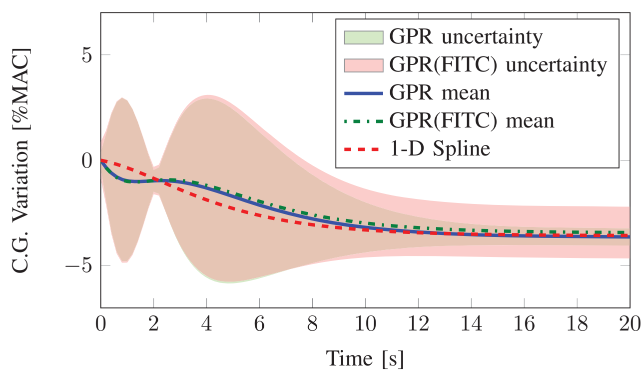

Fig. 5: Estimate of the aircraft longitudinal C.G. variation during level deceleration flight.

It can be seen that the results between the GPR model and the 1-D spline interpolation are pretty close, and the GPR model with FITC approximation gives almost exactly the same results as the full GPR model, with slightly larger uncertainties. Furthermore, the estimate of the GPR model-based method shows significant uncertainties during the first half of the simulation. This is an indication that the mean value from the GPR model needs to be used with caution, since the uncertainty may come from lack of data, low quality of the data available, measurement noises, or even data errors. For this simulation, the uncertainty comes from both the lack of data and measurement noise.

2) Cruise Flight: This scenario lasts for 90 minutes, during which fuel consumption causes the C.G. of the aircraft to shift. The fuel quantity input also has a Gaussian distribution. Simulation results are shown in Fig. 6 and all the results are almost the same.

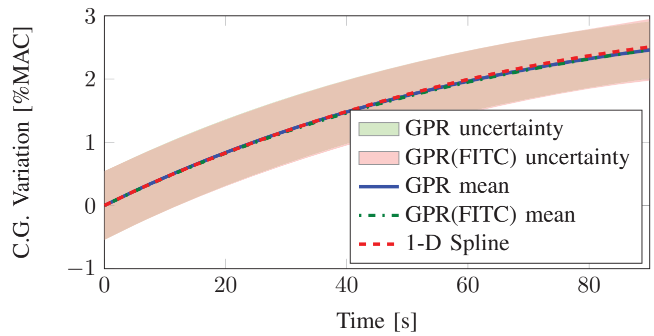

Fig. 6: Estimate of the aircraft longitudinal C.G. variation during cruise flight.

V. Conclusions and Further Research

This paper presents an estimation method for aircraft centre of gravity based on Gaussian process regression models. The proposed method is capable of incorporating uncertainties and provide a probabilistic estimate of the C.G. Numerical examples on a transport aircraft show that the method has small mean squared errors and provides good estimate of the C.G. with additional uncertainty information.

Due to restrictions on the data available, the performance of the proposed method is not extensively investigated. Detailed tests and validation of the method are to be done on a more complete fuel weight property database.

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