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Weighted pseudo-inverse based control allocation of heterogeneous redundant operating mechanisms for DPC aircraft *

Zhihui Wang, Jing Zhang, Lingyu Yang and Xiaoming Guo

Abstract— **Distributed propulsion configuration (DPC) usually have heterogeneous redundant operating mechanisms including redundant control surfaces and thrust vector actuators, thus producing strong and complex control coupling between flight and propulsion systems. Then control allocation of heterogeneous redundant actuators is certainly required to realize integrated control. First, the formulation of this specific control allocation problem is given, and then the Moore-Penrose pseudo-inverse method and weighted pseudo-inverse method are introduced to improve control accuracy and coordinate the complex relationship between different types of actuators. Simulation results reveal that the weighted pseudo-inverse method has higher accuracy in allocation result, and it is suitable for the specific control allocation problem for DPC aircraft.**

Nomenclature

$\mathbf{v}$	virtual control input, $m\times 1$
$\mathbf{u}$	true control input, $n\times 1$
$\mathbf{B}$	control effectiveness matrix, $m\times n$
$\mathbf{P},\mathbf{P_W}$	a generalized inverse of a matrix $\mathbf{B}$, $n\times m$
$\mathbf{u_{min}},\mathbf{u_{max}}$	lower/upper limits of $\mathbf{u}$, $n\times 1$
$\boldsymbol{\lambda}$	an n-vector of LaGrange multipliers, $n\times 1$
$\mathbf{W_u}$	weighting matrix, $n\times n$
$\mathbf{\Phi}$	attainable moment subset, AMS, $\mathbf{\Phi}\subset\mathbf{R}^m$
$\mathbf{err}$	control allocation error, $m\times 1$
ratio	ratio of generalized inverse method AMS to system AMS
pinv	the Moore-Penrose pseudo-inverse method
wpinv	the weighted pseudo-inverse method
$diag(vector)$	a square matrix whose diagonal elements are elements of $vector$, $n\times n$
L,M,N	roll/pitch/yaw moment (J)
T	thrust (N)
δ_{l1},δ_{r2}	left/right inside elevon deflection angle (degree)
δ_{l2},δ_{r2}	left/right middle elevon deflection angle (degree)
δ_{l3},δ_{r3}	left/right outside elevon deflection angle (degree)
δ_{w1},δ_{w2}	left/right winglet rudder deflection angle (degree)
$\pi_{kl}^*,\pi_{km}^*,\pi_{kr}^*$	the pressure ratio of the left/middle/right turbo fan
A_{8l},A_{8m},A_{8r}	the turbo nozzle outlet area of left/middle/right engine (m^2)
$\alpha_{Tl},\alpha_{Tm},\alpha_{Tr},\beta_{Tl},\beta_{Tm},\beta_{Tr}$	thrust vectoring angles of left/middle/right engine in vertical/horizontal direction (degree)

I. Introduction

The adverse impact of the aviation activities on the environment is mainly reflected in noise, air quality, energy consumption and greenhouse gas emissions. To reduce this impact, next-generation civil aircraft development plans are proposed in Europe and the United States, such as SAX-40 and N3-X [1]-[6].

The new aircraft design concept of distributed propulsion configuration (DPC) is adopt by the next-generation civil aircraft. DPC's primary features include: (1) the blended wing body (BWB) layout can significantly improve the lift/drag characteristics [7]; (2) the use of several sets of semi-buried propulsion system can take the initiative to achieve active adjustment of aerodynamic and load distribution. Meanwhile, thrust vector technology and new types of multiple control surfaces, such as ailerons and winglet rudders, are also applied in next-generation civil aircraft. Redundant control surfaces and thrust vector actuators form heterogeneous multiple operating mechanisms, which produce an increased coupling effects between engines and the aircraft. Thus, control allocation is strongly demanded.

The pseudo-inverse method was first used to reconstruct the redundant system when control surfaces failed [8]. The control efficiency matrix directly reflects the performance relationship between the control surface deflection angles and the aerodynamic torques that can be generated [9] [10]. Due to the different speed limits of the aircraft's different control surfaces, the bandwidth is different. Therefore, different control surfaces should be designed with different weights [11]. The weighted pseudo-inverse method improves the pseudo-inverse method under this idea.

In this paper, a specific DPC aircraft is selected as the research object. First, the characteristics of DPC aircraft and the formulation of control allocation to be solved are described in the form of mathematics. Then, the Moore-Penrose pseudo-inverse method is analyzed and proved with the optimal target of lowest energy consumption. To improve the reliability, coordinate the relationship between various surfaces and ensure the lowest energy consumption at the same time, weighted pseudo-inverse method is proposed and applied. Finally, the results of these two methods are given and analyzed by simulation.

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I. Description of DPC aircraft and control allocation

A. Description of DPC aircraft

As shown in Figure 1. , The DPC aircraft has six elevons and two winglet rudders at the edge of wings and also three engines with nine fans at the end of fuselage. It has no horizontal tails. These features produce strong coupling effect in roll and yaw channels, which makes control more difficult. These three engines adopt two-dimensional (2-D) thrust vector technology, and each engine has a variable exhaust nozzle at the duct exits. Meanwhile, fan pressure ratio of engines is variable.

B. Formulation of control allocation

The control allocation problem is to distribute a desired total control effort, such as pitch/roll/yaw moment, and thrust of engines among a redundant set of actuators [9]. It is formulated as follows.

Given $\mathbf{B}, \mathbf{v}, \mathbf{u}_{\min}$ and $\mathbf{u}_{\max}$, find $\mathbf{u}$ such that,

$$\mathbf{Bu} = \mathbf{v}, \quad \mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max} \tag{1}$$

where

$$\begin{aligned}
&\mathbf{v} = [L, M, N, thrust]^T, \\
&\mathbf{u} = [\delta_{l1}, \delta_{r2}, \delta_{l2}, \delta_{r2}, \delta_{l3}, \delta_{r3}, \delta_{w1}, \delta_{w2}, \\
&\quad \pi^*_{kl}, A_{8l}, \pi^*_{km}, A_{8m}, \pi^*_{kr}, A_{8r}, \\
&\quad \alpha_{Tl}, \beta_{Tl}, \alpha_{Tm}, \beta_{Tm}, \alpha_{Tr}, \beta_{Tr}]^T, \\
&\begin{cases}
-60^\circ \le \delta_{l1}, \delta_{r2}, \delta_{l2}, \delta_{r2}, \delta_{l3}, \delta_{r3}, \delta_{w1}, \delta_{w2}, \le 60^\circ \\
1 \le \pi^*_{kl}, \pi^*_{km}, \pi^*_{kr} \le 1.6 \\
1 \le A_{8l}, A_{8m}, A_{8r} \le 1.6 \\
-30^\circ \le \alpha_{Tl}, \beta_{Tl}, \alpha_{Tm}, \beta_{Tm}, \alpha_{Tr}, \beta_{Tr} \le 30^\circ
\end{cases}
\end{aligned} \tag{2}$$

Equation (1) can be considered as a linear mapping under the control effectiveness matrix $\mathbf{B}$

$$\mathbf{B} : \mathbf{R}^n \to \mathbf{R}^m \tag{3}$$

However, $\mathbf{B}$ may change according to the vector $\mathbf{u}$, but it is fixed during every simulation step.

The space of (1) is marked as subset $\mathbf{\Omega}$,

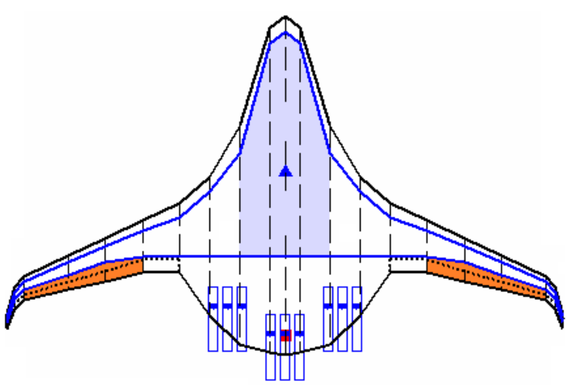

Figure 1. Control surfaces configuration of DPC aircraft [2]

$$\mathbf{\Omega} = \left\{ \mathbf{u} \in \mathbf{R}^n \middle| \mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max} \right\} \subset \mathbf{R}^n \tag{4}$$

Under the linear mapping (3), $\mathbf{\Phi}$ is defined as follows

$$\mathbf{B} : \mathbf{\Omega} \to \mathbf{\Phi} \tag{5}$$

where $\mathbf{\Phi} \subset \mathbf{R}^m$ is called attainable moment subset (AMS): a subset of all moments [12].

The set of (1) is under-determined (fewer equations than unknowns) and mathematically has an infinite number of solutions. When control limitations are introduced, the equations may have no solutions [13].

II. Control allocation based on generalized inverse

How to find the best solution of (1) is a realistic problem, especially when there are infinitely many solutions. In this section, pseudo-inverse method is introduced and proved. All the control variables participate in the control during the entire flight. Therefore, the total deflection of the aerodynamic control surfaces can be reduced. However, $\mathbf{u}$ contains different kinds of control inputs. It is unreasonable to treat them equally. For this reason, weighted pseudo-inverse method is adopted. This method minimizes the 2-norm of the vector $\mathbf{W_u u}$. Obviously, additional fuel is required. But it is flexible when we want to adjust the deflection of actuators during different flight phases and under different mission requirements by changing weighting matrix $\mathbf{W_u}$.

A. The Moore-Penrose pseudo-inverse method

Matrix $\mathbf{P}$ is a particular generalized inverse of the matrix $\mathbf{B}$ which minimizes the 2-norm of our control input vector $\mathbf{u}$ when solving the problem of control allocation. The 2-norm of a vector is just the positive square-root of the sum of the squares of the individual controls, known as the length of the vector. This inverse $\mathbf{P}$ is called the Moore–Penrose pseudo-inverse.

When matrix $\mathbf{B}$ is full rank and the limits $\mathbf{u}_{\min}, \mathbf{u}_{\max}$ are ignored, the formula (1) has an infinite number of solutions. We can find the only one with the Moore–Penrose pseudo-inverse method.

And the true control input is

$$\mathbf{u} = \mathbf{B}^{\mathrm{T}} \left(\mathbf{B}\mathbf{B}^{\mathrm{T}} \right)^{-1} \mathbf{v} = \mathbf{Pv} \tag{6}$$

where $\mathbf{P} = \mathbf{B}^{\mathrm{T}} \left(\mathbf{B}\mathbf{B}^{\mathrm{T}} \right)^{-1}$.

It is proved as follows.

We define the scalar function using LaGrange multipliers

$$\mathrm{H}(\mathbf{u}, \boldsymbol{\lambda}) = \frac{1}{2} \mathbf{u}^{\mathrm{T}} \mathbf{u} + \boldsymbol{\lambda}^{\mathrm{T}} (\mathbf{v} - \mathbf{Bu}) \tag{7}$$

The factor of $1/2$ is used to eliminate the factor of 2. H will be a minimum (or maximum) when

$$\frac{\partial \mathrm{H}}{\partial \mathbf{u}} = \mathbf{0}, \frac{\partial \mathrm{H}}{\partial \boldsymbol{\lambda}} = \mathbf{0} \tag{8}$$

Performing the operations will yield

$$\frac{\partial \mathrm{H}}{\partial \mathbf{u}} = \mathbf{u}^{\mathrm{T}} - \boldsymbol{\lambda}^{\mathrm{T}} \mathbf{B} = \mathbf{0} \tag{9}$$

Hence, we require that $\mathbf{u}^{\mathrm{T}} = \boldsymbol{\lambda}^{\mathrm{T}} \mathbf{B}$, or $\mathbf{u} = \mathbf{B}^{\mathrm{T}} \boldsymbol{\lambda}$

$$\frac{\partial \mathrm{H}}{\partial \boldsymbol{\lambda}} = \mathbf{v} - \mathbf{Bu} = \mathbf{0} \tag{10}$$

So that, $\mathbf{v} = \mathbf{Bu}$. Now combing the two results

$$\mathbf{v} = \mathbf{Bu} = \mathbf{BB}^{\mathrm{T}}\boldsymbol{\lambda} \tag{11}$$

Since $\mathbf{B}$ is full rank, $\mathbf{B}^{\mathrm{T}}\mathbf{B}$ is too, and since $\mathbf{B}^{\mathrm{T}}\mathbf{B}$ is square, it is invertible. Thus

$$\boldsymbol{\lambda} = \left(\mathbf{BB}^{\mathrm{T}}\right)^{-1}\mathbf{v} \tag{12}$$

Since $\mathbf{u} = \mathbf{B}^{\mathrm{T}}\boldsymbol{\lambda}$ we have

$$\mathbf{u} = \mathbf{B}^{\mathrm{T}}\left(\mathbf{BB}^{\mathrm{T}}\right)^{-1}\mathbf{v} = \mathbf{Pv} \tag{13}$$

where $\mathbf{P} = \mathbf{B}^{\mathrm{T}}\left(\mathbf{BB}^{\mathrm{T}}\right)^{-1}$. □

The principal claim made about the minimum-norm pseudo-inverse is that because it minimizes the sum of the squares of the control effector displacements, it thus minimizes the consumption of energy of all actuators.

B. Weighted pseudo-inverse method

The Moore–Penrose inverse is just one of inverses that minimizes a vector norm. Entire families of these solutions can be obtained from optimization problems that aim to minimize other norms of $\mathbf{u}$. The weighted 2-norm is used frequently. And it minimizes $\mathbf{u}^{\mathrm{T}}\mathbf{W}_{\mathbf{u}}^{\mathrm{T}}\mathbf{W}_{\mathbf{u}}\mathbf{u}$, where $\mathbf{W}_{\mathbf{u}}$ is a positive diagonal matrix.

The solution of (1) using weighted pseudo-inverse method is given

$$\mathbf{u} = \mathbf{W}_{\mathbf{u}}^{-1}\mathbf{B}^{\mathrm{T}}\left(\mathbf{B}\mathbf{W}_{\mathbf{u}}^{-1}\mathbf{B}^{\mathrm{T}}\right)^{-1}\mathbf{v} = \mathbf{P}_{\mathbf{w}}\mathbf{v} \tag{14}$$

where $\mathbf{P}_{\mathbf{w}} = \mathbf{W}_{\mathbf{u}}^{-1}\mathbf{B}^{\mathrm{T}}\left(\mathbf{B}\mathbf{W}_{\mathbf{u}}^{-1}\mathbf{B}^{\mathrm{T}}\right)^{-1}$.

We can easily get (14) by replacing $\mathbf{u}$ of $\mathbf{W}_{\mathbf{u}}\mathbf{u}$ from (7) ~ (13). □

Regarding the issue above, the Moore-Penrose pseudo-inverse method and weighted pseudo-inverse method can be used to deal with control allocation problem with their own optimal indexes.

III. Simulation

In this section, a special DPC aircraft is chosen with parameters and flight state point data shown in TABLE I.

Now consider a specially chosen virtual control input $\mathbf{v} = [-10435529, 6147752, -707065, 265852]^T$.

TABLE I. GEOMETRIC PARAMETERS AND FLIGHT DATA OF A SPECIAL DPC AIRCRAFT

Parameter	Value
Wing area, m^2	836
Wing span, m	63.22
C.G., % centerbody chord	58.3
Maximum take-off weight, Kg	150,847
Flight height, m	5,000
Mach	0.6
Angle of attack for trim, degree	1.72

For the convenience of recording, we take the control effectiveness matrix $\mathbf{B}_{pinv} = [\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3, \mathbf{B}_4]$,

$$\mathbf{B}_1 = \begin{bmatrix} 9.8951E5 & -9.8951E5 & 1.1050E6 & -1.1050E6 \\ -5.6287E5 & -5.6287E5 & -3.6253E5 & -3.6253E5 \\ 3.7672E3 & -3.7672E3 & 2.3859E4 & -2.3859E4 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_2 = \begin{bmatrix} 1.2206E6 & -1.2206E6 & -7.0320E3 & -7.0320E3 \\ -1.6220E5 & -1.6220E5 & 8.7814E3 & 8.7814E3 \\ 2.3859E4 & -2.3859E4 & -1.0284E5 & -1.0284E5 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 235700 & 46180 & 235700 & 46180 & 23570 & 46180 \end{bmatrix},$$

$$\mathbf{B}_4 = \begin{bmatrix} -1.0827E4 & 0 & 0 & 0 & 1.0827E4 & 0 \\ 2.6989E4 & 0 & 2.6989E4 & 0 & 2.6989E4 & 0 \\ 0 & 2.6989E4 & 0 & 2.6989E4 & 0 & 2.6989E4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

If we ignore the small difference between $\mathbf{B}_{pinv}$ and $\mathbf{B}_{wpinv}$, they are approximately equal. In order to make full use of the elevons and winglet rudders and make the difference between π_k^* and A_8 , we choose

$$\begin{aligned}\mathbf{W}_{\mathbf{u}} = diag([&1/60, 1/60, 1/60, 1/60, 1/60, 1/60, 1/60, 1/60, \\ &1/10, 1/21, 1/10, 1/21, 1/10, 1/21, \\ &1/25, 1/25, 1/25, 1/25, 1/25, 1/25]) \end{aligned}.$$

On the one hand, considering that the range of control input $\mathbf{u}$ change is not symmetrical about the origin point, we choose the equilibrium position $\mathbf{u}_0 = [0,0,0,0,0,0,0,0,1.3,1.3,1.3,1.3,1.3,1.3,0,0,0,0,0,0]^T$. On the other hand, when calculating matrix $\mathbf{B}$, a constant term $\mathbf{v}_f$ is introduced into (1) . so,

$$\mathbf{B}\left(\mathbf{u} + \mathbf{u}_0\right) = \mathbf{v} - \mathbf{v}_f \tag{15}$$

We can easily calculate the solution of the system, according to (13) , (14) and (15), see TABLE II.

$$\begin{aligned}\mathbf{u}_{pinv} = [&-4.74, -2.49, -4.02, -0.63, -2.86, 0.77, 2.35, 2.35, \\ &1.43, 1.33, 1.43, 1.33, 1.43, 1.33, 0.18, -0.60, 0.17, -0.60, 0.16, -0.60]^T,\end{aligned}$$

TABLE II. RESULTS OF CONTROL ALLOCATION USING TWO METHODS

Parameter	pinv	wpinv
True control input	$\mathbf{u}_{\mathbf{pinv}}$	$\mathbf{u}_{\mathbf{wpinv}}$
2-norm of $\mathbf{u}$	8.81	8.94
2-norm of $\mathbf{W}_{\mathbf{u}}\mathbf{u}$	--	0.32
AMS	See Fig. 2	See Fig. 3
ratio	53.23 %	49.76%
err	$\mathbf{err}_{\mathbf{pinv}}$	$\mathbf{err}_{\mathbf{pinv}}$
$\lVert\mathbf{err}\rVert_2$	1.6632	1.4560
maximum error	1.4580	1.4248
number of iterations	20	20
average running time, s	0.99	0.95

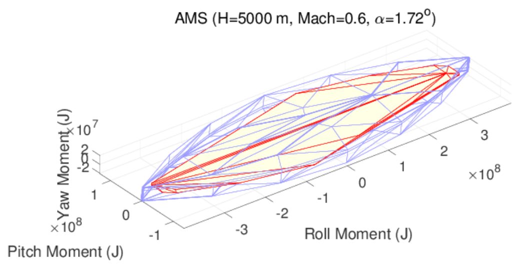

Figure 2. AMS with the Moore-Penrose pseudo-inverse method

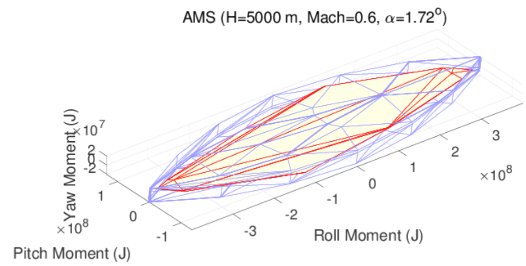

Figure 3. AMS with the weighted pseudo-inverse method

$$\mathbf{u}_{wpinv} = [-4.86, -2.64, -4.12, -0.72, -2.91, 0.74, 2.48, 2.48, \\ 1.42, 1.40, 1.42, 1.40, 1.42, 1.40, 0.08, -0.26, 0.08, -0.26, 0.07, -0.26]^T$$

$$\mathbf{err_{pinv}} = [-0.0146, 0.7883, -0.1371, -1.4580]^T,$$

$$\mathbf{err_{wpinv}} = [1.0625E-5, 0.0640, -0.2930, -1.4248]^T$$

It is found that elevon deflection angles, including $\delta_{l1}, \delta_{r2}, \delta_{l2}, \delta_{r2}, \delta_{l3}, \delta_{w1}, \delta_{w2}$, are increased fully comparing $\mathbf{u}_{wpinv}$ with $\mathbf{u}_{pinv}$. Roll and pitch moments are compensated by the reduce of the rest of elevon angles and thrust vectoring angles $\alpha_{Tl}, \alpha_{Tm}, \alpha_{Tr}, \beta_{Tl}, \beta_{Tm}, \beta_{Tr}$. In order to keep the thrust constant, the change trend of π_k^* is opposite to that of A_8. Aerodynamic surfaces are used more effectively than before. This is consistent with the principle of using as little thrust vector as possible due to its limited service life.

From matrix $\mathbf{B}$, we can see that thrust and moments are completely decoupled. So, we delete the forth row of $\mathbf{B}$ and use the rest elements to obtain AMS, see Fig. 2 and Fig. 3.

The volume formed by the red line donates AMS using generalized inverse method, while blue section donating system AMS. Ratio becomes smaller from 53.23% to 49.76%.

IV. CONCLUSIONS

The new design concept of DPC and heterogeneous multiple operating mechanisms redundancy expand control space, and they also strengthen the coupling effects between the flight and propulsion systems. To solve the problem caused by strong coupling and operating mechanisms redundancy, control allocation is needed. This paper utilizes the weighted pseudo-inverse method to achieve control allocation with the minimum fuel energy consumption. However, the weighted matrix is fixed during the entire simulation. Next work is to automatically adjust $\mathbf{W_u}$ online.

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