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IEEE Control Systems Chapter, Singapore

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IEEE ICCA 2020 Program Overview

October 9-11, 2020

(Based on Singapore Time)

DAY 1 (October 9, Friday), Zoom Passcode: See Email Notice		
8:30-9:00 (Singapore time)	**Opening session** (https://ntu-sg.zoom.us/j/98233820953) (https://meeting.tencent.com/s/DnQuOoxY2J1N)	
9:00-10:00	**Keynote by Prof. Christos G. Cassandras** **Title: Bridging the Gap between Optimal Control and Provably Safe Real-Time Control: Theory and Applications to Autonomous Vehicles** (https://ntu-sg.zoom.us/j/98176669600) (https://meeting.tencent.com/s/XW7DjVVV2uVJ)	
10:15-11:15	**Keynote by Prof. Jie Huang** **Title: The Cooperative Output Regulation of Multi-agent Systems: An Integrated Approach** (https://ntu-sg.zoom.us/j/91082157627) (https://meeting.tencent.com/s/QfdX6mnLasFR)	
11:30-12:30	**Keynote by Prof. Robert Bitmead** **Title: Coding Aspects of Control** (https://ntu-sg.zoom.us/j/93612535076) (https://meeting.tencent.com/s/BUT97Yl5IFzr)	
	Lunch Break	
13:30-15:10	**Best Paper Award Session** Papers: 76, 102, 104, 228, 285 (https://ntu-sg.zoom.us/j/98176669600) (https://meeting.tencent.com/s/V6V80iWLK5Cx)	**Best Student Paper Award Session** Papers: 33, 277, 295, 350, 441 (https://ntu-sg.zoom.us/j/95590272355) (https://meeting.tencent.com/s/V6bPcUGWL5pb)
15:30-17:00	**Session FriA1 - Advanced Control Methods and Applications in Autonomous Systems** Papers: 50, 95, 184, 236, 430, 434 (https://ntu-sg.zoom.us/j/94968922376)	**Session FriA2 - Advances in Control of Network Systems** Papers: 70, 75, 123, 137, 261 (https://ntu-sg.zoom.us/j/92028781822)
	Session FriA3 - Advanced Control of Multi-Agent Systems Papers: 68, 80, 114, 317, 353, 421 (https://ntu-sg.zoom.us/j/96553477502)	**Session FriA4 - Robotics 1** Papers: 61, 79, 86, 126, 195, 386 (https://ntu-sg.zoom.us/j/99507005343)
	Session FriA5 - Cooperative Control and Optimization of Network Systems Papers: 109, 152, 246, 368, 375 (https://ntu-sg.zoom.us/j/91477169933)	**Session FriA6 - Control Applications 1** Papers: 101, 267, 337, 367, 420 (https://ntu-sg.zoom.us/j/99003929275)
17:15-18:45	**Session FriB1 - Attack Detection and State Estimation in Cyber-Physical Systems** Papers: 180, 323, 325, 401, 407 (https://ntu-sg.zoom.us/j/91500226291)	**Session FriB2 - Filtering & Signal Processing** Papers: 67, 163, 348, 378, 383, 408 (https://ntu-sg.zoom.us/j/97981586117)
	Session FriB3 - Autonomous Control for Unmanned Aerial Vehicle Papers: 192, 342, 344, 376, 382 (https://ntu-sg.zoom.us/j/92348681455)	**Session FriB4 - Robotics 2** Papers: 127, 211, 345, 393, 406 (https://ntu-sg.zoom.us/j/91514675079)

	Session FriB5 - Control, Optimization and Its Applications Papers: 247, 248, 249, 278, 311 (https://ntu-sg.zoom.us/j/98079682625)	**Session FriB6 - Control Applications 2** Papers: 98, 183, 237, 309, 343, 431 (https://ntu-sg.zoom.us/j/95654530446)

DAY 2 (October 10, Saturday), Zoom Passcode: See Email Notice		
9:00-10:00 (Singapore time)	**Keynote by Prof. Jing Sun** **Title: Predictive Control of Integrated Power and Thermal Management for Electrified and Connected Transportation Platforms** (https://ntu-sg.zoom.us/j/93984550138) (https://meeting.tencent.com/s/WBiPoPmKVgmg)	
10:15-11:15	**Keynote by Prof. Ian Petersen** **Title: Risk Sensitive Control for Quantum Linear Systems** (https://ntu-sg.zoom.us/j/91809751205) (https://meeting.tencent.com/s/Er0sHZht7aCY)	
11:30-12:30	**Keynote by Prof. Ji-Feng Zhang** **Title: How to Use Less Data to Get Good Estimates and Controls** (https://ntu-sg.zoom.us/j/98001096527) (https://meeting.tencent.com/s/hazIiuHHF7rw)	
	Lunch Break	
13:30-15:00	**Session SatA1 - Recent Advances in Cooperative Control of Swarm Systems** Papers: 64, 83, 162, 207, 279, 365 (https://ntu-sg.zoom.us/j/94667765334)	**Session SatA2 - Mult-Agent System & Distributed Design 1** Papers: 48, 186, 226, 231, 234, 338 (https://ntu-sg.zoom.us/j/99484372028)
	Session SatA3 - Intelligent sensing, control, estimation and their applications Papers: 54, 81, 136, 150, 201, 391 (https://ntu-sg.zoom.us/j/97281714570)	**Session SatA4 - Distributed computing and control of complex systems** Papers: 74, 117, 149, 154, 194 (https://ntu-sg.zoom.us/j/93127207064)
	Session SatA5 - Learning System & Optimal Control 2 Papers: 219, 232, 233, 240, 271, 272 (https://ntu-sg.zoom.us/j/96942843275)	**Session SatA6 - Cooperative Guidance and Control of Multiple Autonomous Vehicles** Papers: 169, 326, 360, 412, 438 (https://ntu-sg.zoom.us/j/95372491730)
15:15-16:45	**Session SatB1 - Distributed coordination and optimization for multi-agent systems** Papers: 82, 112, 148, 327, 377 (https://ntu-sg.zoom.us/j/99269861413)	**Session SatB2 - Mult-Agent System & Distributed Design 2** Papers: 47, 168, 172, 3333, 390, 440 (https://ntu-sg.zoom.us/j/92154040535)
	Session SatB3 - Information fusion and control in unmanned systems 1 Papers: 281, 283, 286, 302, 318, 437 (https://ntu-sg.zoom.us/j/95925474930)	**Session SatB4 - Intelligent control and estimation for hybrid complex systems** Papers: 49, 93, 196, 266, 282, 399 (https://ntu-sg.zoom.us/j/95599504149)
	Session SatB5 - Learning System & Optimal Control 3 Papers: 52, 328, 389, 395, 404, 411 (https://ntu-sg.zoom.us/j/98310438397)	**Session SatB6 - Frontiers in Finite-Valued Networked Systems** Papers: 96, 111, 212, 341, 358 (https://ntu-sg.zoom.us/j/94150191160)
	Session SatC1 - Information fusion and control in unmanned systems 2 Papers: 216, 217, 238, 268, 270, 276 (https://ntu-sg.zoom.us/j/98088965445)	**Session SatC2 - Frontiers in Distributed Control and Optimization** Papers: 108, 210, 215, 312, 351 (https://ntu-sg.zoom.us/j/99590352629)

17:00-18:30	**Session SatC3 - Learning System & Optimal Control 1** Papers: 51, 59, 107, 171, 178, 356 (https://ntu-sg.zoom.us/j/94109310417)	**Session SatC4 - Distributed Optimization and Learning** Papers: 103, 160, 214, 332, 387 (https://ntu-sg.zoom.us/j/97252910824)
	Session SatC5 - Intelligent Sensing and Safe Control System Technologies of Unmanned Aerial Vehicle Papers: 147, 176, 355, 435, 436, 439 (https://ntu-sg.zoom.us/j/95562539552)	**Session SatC6 - Nonlinear Control** Papers: 57, 69, 72, 91, 220, 423 (https://ntu-sg.zoom.us/j/94013707778)

DAY 3 (October 11, Sunday), Zoom Passcode: See Email Notice		
9:00-9:50 (Singapore time)	**Award Ceremony** (https://ntu-sg.zoom.us/j/91462345017) (https://meeting.tencent.com/s/wDou9FecLphw)	
10:00-11:40	**Plenary Panel Discussion Session, by Profs. Clarence W. de Silva, Wei Kang, Tong Boon Quek, Yacov A. Shamash, Bin Xin, and Chaired by Profs. Jie Chen and Ben M. Chen** **Topic: Autonomous Systems, AI and Commercialization Opportunities** (https://cuhk.zoom.us/j/97039359394) (https://meeting.tencent.com/s/VE8ClJZEhh2q)	
	Lunch Break	
13:00-14:30	**Session SunA1 - Robotic Manipulation and Coordination of Multi-Robot Systems** Papers: 202, 245, 303, 339, 354, 416 (https://ntu-sg.zoom.us/j/96783107921)	**Session SunA2 - On distributed control of multi-agent systems** Papers: 42, 63, 159, 161, 340, 388 (https://ntu-sg.zoom.us/j/96220596383)
	Session SunA3 - Sensing, Estimation, Control and Optimization for Networked Systems Papers: 177, 193, 223, 235, 255, 335 (https://ntu-sg.zoom.us/j/96429857632)	**Session SunA4 - Robotics 3** Papers: 134, 257, 366, 413, 415, 433 (https://ntu-sg.zoom.us/j/91967328904)
	Session SunA5 - Modeling, Control and Optimization of Cyber-Physical Systems Papers: 110, 113, 153, 205, 253, 319 (https://ntu-sg.zoom.us/j/95297276240)	**Session SunA6 - Recent Advances in Control of Networked Systems** Papers: 92, 106, 142, 170, 182, 299 (https://ntu-sg.zoom.us/j/97455636411)
14:45-16:15	**Session SunB1 - On Cyber-physical Systems: Control, Optimization and Security** Papers: 90, 132, 135, 155, 315 (https://ntu-sg.zoom.us/j/94926726246)	**Session SunB2 - Recent advances on distributed control and optimization methods for networked systems** Papers: 129, 130, 141, 151, 274, 369 (https://ntu-sg.zoom.us/j/91537640115)
	Session SunB3 - Integrated Networked Control for Massively Many IoT Devices Papers: 187, 204, 229, 384, 419 (https://ntu-sg.zoom.us/j/92391155037)	**Session SunB4 - Unmanned Systems: Theory and Applications** Papers: 119, 181, 190, 275, 307 (https://ntu-sg.zoom.us/j/93651629908)
	Session SunB5 - Observation, Disturbance Rejection, and Learning for uncertain systems Papers: 143, 145, 330, 371, 381 (https://ntu-sg.zoom.us/j/96650156404)	**Session SunB6 - The Coordinated Control of Multi-Agent Systems** Papers: 84, 124, 296, 370, 379 (https://ntu-sg.zoom.us/j/92551965135)

16:30-18:00	**Session SunC1 - Control and Estimation for Stochastic Systems** Papers: 158, 197, 198, 244, 254, 289 (https://ntu-sg.zoom.us/j/99786533327)	**Session SunC2 - Optimal Control of Uncertain Systems and Its Applications** Papers: 144, 262, 300, 313, 361 (https://ntu-sg.zoom.us/j/97986102911)
	Session SunC3 - Dynamic Modeling and Control in Biomechanics and Biomedical Engineering Papers: 121, 122, 140, 206, 225 (https://ntu-sg.zoom.us/j/92383091765)	**Session SunC4 - Sliding Mode Control** Papers: 20, 88, 189, 269, 324 (https://ntu-sg.zoom.us/j/93816857523)
	Session SunC5 - System Modelling & Identification Papers: 24, 156, 157, 175, 224 (https://ntu-sg.zoom.us/j/95064337875)	**Session SunC6 - Fault Detection & Neural System** Papers: 19, 36, 185, 251, 357 (https://ntu-sg.zoom.us/j/98429623072)
18:15-18:30	**Closing** (https://ntu-sg.zoom.us/j/99170239742) (https://meeting.tencent.com/s/4AwxFUwIaGAe)	

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Minimum-energy Control of Nearly-controllable Discrete-time Bilinear Systems with Nondiagonalizable Structure

Linxiang Cheng, Jiapeng Liu, and Lin Tie

Abstract— **In this paper, a class of discrete-time bilinear systems which are uncontrollable but can be nearly-controllable is considered. A necessary and sufficient condition for near-controllability of such systems was obtained and the corresponding control inputs to achieve the transition of the systems between any given pair of states are computable, but there are infinite groups of such control inputs. Therefore, this paper proposes a new algorithm, which attempts to find a minimum-energy control sequence to achieve the state transition and improves the previous algorithm for computing the control inputs. Analysis and examples are given to illustrate the proposed algorithm.**

I. INTRODUCTION

Near-controllability is defined for those nonlinear systems that are uncontrollable but own a large controllable region [1,2], which has recently attracted attention [3,4]. The interest of near-controllability lies in the fact that it can not only better characterize nonlinear systems but also prove controllability [5,6]. More specifically, there do exist uncontrollable nonlinear systems which are nearly-controllable, and if we only use "uncontrollable" to describe such systems, we may miss some valuable properties of them. Furthermore, a controllable system is also nearly-controllable and it is easier to prove near-controllability than controllability. When the near-controllability problem of a control system is solved, it is not far from proving its controllability.

The notion of near-controllability was first introduced and demonstrated on the following discrete-time bilinear system

$$x\left(k+1\right)=\left(I+u\left(k\right)B\right)x\left(k\right), \tag{1}$$

where $x\left(k\right)\in\mathbb{R}^n$, $u\left(k\right)\in\mathbb{R}$, and $B\in\mathbb{R}^{n\times n}$ is a constant matrix. Bilinear systems have been extensively investigated over decades. Such systems form an important class of nonlinear systems, which have wide applications ranging from engineering to non-engineering fields, e.g. chemistry, biology, and socio-economics [7,8]. For instance, system (1) can be used to model the biological species populations, where $x\left(k\right)$ is the population that needs to be positive and $u\left(k\right)$ is the growth rate that is constrained. Moreover, the applications of discrete-time bilinear systems have increased in the modeling and control of power systems [9] and complex networks [10,11].

Since system (1) is uncontrollable on $\mathbb{R}^n$ for any finite dimension[1], it is natural to consider the largest controllable region[2] of the system in the state space. As a result, the near-controllability problem of system (1) was considered and a necessary and sufficient condition for system (1) with B having only real eigenvalues to be nearly-controllable was obtained in [1]. Specifically, [1] proved that, under the condition of B's eigenvalues being nonzero real and B being cyclic and having no Jordan block with dimension greater than two in its Jordan canonical form, the uncontrollable system (1) has a large controllable region which nearly covers $\mathbb{R}^n$ such that the system is nearly-controllable. Furthermore, thanks to the root locus approach, the control inputs to achieve state transition between any given pair of states can be computed. The similar idea was also used in [12] to prove near-controllability of discrete-time bilinear systems with diagonal form and to obtain the computable control inputs, where the corresponding transfer function derived in [12] contains only single poles due to the diagonal form, so that all the root loci start moving along the real axis no matter what the zeros are and the Implicit Function Theorem was not needed in [12]. This is different from the nondiagonal case in [1], where the root locus approach and the Implicit Function Theorem are both needed and near-controllability is more difficult to prove.

Although [1] completely solved the near-controllability problem of system (1) when B has only real eigenvalues and the required control inputs to achieve state transition can be computed, the optimal control problem, especially, the minimum-energy control problem has not been well addressed. Note that in the diagonal case, i.e. B is diagonalizable, the minimum-time and minimum-energy control problems were studied in [13,14] by derivation analysis of single variable and the root locus theory, and theoretical algorithms were proposed to prove the optimality. However, if B is nondiagonalizable, the minimum-energy control problem would be much more complicated since there are more parameter variables and the algorithms proposed in [13,14] do not work. Therefore, in this paper, we aim to

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[1] Although the system is controllable on $\mathbb{R}^n_*:\mathbb{R}^n\backslash\{0\}$ for $n=1,2$, it is uncontrollable on $\mathbb{R}^n_*$ for any $n>2$, let alone $\mathbb{R}^n$.

[2] A controllable region is a region in the state space on which the system is controllable. Namely, for any ξ,η in this region, there exist control inputs that steer the system from ξ to η.

study the minimum-energy control problem of system (1) with nondiagonalizable structure, i.e. B is nondiagonalizable. Based on the original algorithm in [1] for computing (not minimum-energy) control inputs, we propose a new algorithm to try to solve the minimum-energy control inputs in the nondiagonalizable structure case.

The paper is organized as follows. The near-controllability theory and minimum-energy control problem for systems (1) are presented in Section II. An algorithm for solving the minimum-energy control inputs is proposed in Section III. Examples are provided in Section IV and concluding remarks are given in Section V.

II. PROBLEM FORMULATION

Definition 1 [1]. System (1) is said to be *nearly-controllable* if, for any $\xi \in \mathbb{R}^n \setminus \mathcal{E}$ and any $\eta \in \mathbb{R}^n \setminus \mathcal{F}$, there exist a finite control sequence $u(k)$ $(k = 0, 1, \ldots, l-1)$ where l is a positive integer such that the system can be steered from ξ to η at $k = l$, where $\mathcal{E}$ and $\mathcal{F}$ are two sets of zero Lebesgue measure in $\mathbb{R}^n$.

If $\mathcal{E}$, $\mathcal{F} = \emptyset$, the definition degenerates to the controllability definition. That is, near-controllability includes the notion of controllability and is defined more generally than controllability. This property is well-suited and valuable to analyze those systems that are uncontrollable but have a large controllable region from the perspective of controllability. For near-controllability of system (1), [1] gave a necessary and sufficient condition as follows.

Theorem 1 [1]. Consider system (1) with B having only real eigenvalues. Then, the system is nearly-controllable if and only if B is nonsingular, cyclic, and has no Jordan block with dimension greater than two in its Jordan canonical form.

Note that, if B satisfies the conditions in Theorem 1 and if it is also nondiagonalizable, then B can be transformed into the following form

$$\begin{bmatrix} \lambda_1 & 1 & & & & & & \\ & \lambda_1 & & & & & & \\ & & \ddots & & & & & \\ & & & \lambda_r & 1 & & & \\ & & & & \lambda_r & & & \\ & & & & & \lambda_{r+1} & & \\ & & & & & & \ddots & \\ & & & & & & & \lambda_m \end{bmatrix}, \tag{2}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_m$ are nonzero, real and pairwise distinct and $m + r = n$. Thus, without loss of generality, B is assumed to take the form in (2) throughout this paper.

Based on the proof of Theorem 1, [1] also proposed an algorithm to solve the control inputs steering the nearly-controllable system (1) from given initial state to given terminal state in

$$\mathbb{R}^n \setminus \left\{ \xi \,\middle|\, \left| \begin{matrix} \xi & B\xi & \cdots & B^{n-1}\xi \end{matrix} \right| = 0 \right\}.$$

Algorithm 1 [1]. Steps on computing control inputs for given initial state ξ and terminal state η.

Step 1. Transform B into the Jordan canonical form by a nonsingular matrix P. ξ, η are thus transformed into $P\xi$, $P\eta$, respectively.

Step 2. Find the control inputs that transfer $P\xi$ to a state ζ which belongs to the same orthant as $P\eta$ belongs to.

Step 3. Get the transition matrix $T_{\zeta \to P\eta}$ for ζ, $P\eta$

$$T_{\zeta \to \eta} \triangleq \begin{bmatrix} T_1 & & & & & \\ & \ddots & & & & \\ & & T_r & & & \\ & & & \frac{\eta_{2r+1}}{\zeta_{2r+1}} & & \\ & & & & \ddots & \\ & & & & & \frac{\eta_n}{\zeta_n} \end{bmatrix},$$

where $T_i \triangleq \begin{bmatrix} \frac{\eta_{2i}}{\zeta_{2i}} & \frac{\eta_{2i-1}}{\zeta_{2i}} - \frac{\zeta_{2i-1}\eta_{2i}}{\zeta_{2i}^2} \\ 0 & \frac{\eta_{2i}}{\zeta_{2i}} \end{bmatrix}$.

Step 4. Choose λ_{m+1}, λ_{m+2} such that

$$\begin{aligned} 0 < |\lambda_{m+1}| < \min\{|\lambda_1|, \ldots, |\lambda_m|\} < \\ \max\{|\lambda_1|, \ldots, |\lambda_m|\} < -\lambda_{m+2}, \end{aligned} \tag{3}$$

where $\lambda_1, \ldots, \lambda_m$ are the eigenvalues of B.

Step 5. Choose a positive integer q and compute $T_{\zeta \to P\eta}^{\frac{1}{q}}$.

Step 6. Obtain the root loci of

$$\begin{aligned} & 1 + KG(s) \triangleq 1 + \\ & \frac{K\left((-1)^{2m+2}\mu_1 s^{2m+2} + \cdots + (-1)\mu_{2m+2}s + 1\right)}{s(s+\lambda_1)^2 \cdots (s+\lambda_m)^2 (s+\lambda_{m+1})(s+\lambda_{m+2})} = 0, \end{aligned} \tag{4}$$

where $\begin{bmatrix} \mu_1 & \cdots & \mu_{2m+2} \end{bmatrix}^T$ is given in (2.15) in [1].

If any of the root loci leaves the real axis directly at the pole, then return to the former step and choose another integer q greater than the previous chosen one. Otherwise, choose a suitable K such that the roots of $1 + KG(s) = 0$ are all real. Then, the reciprocals of the real roots of $1 + KG(s) = 0$ are the control inputs that transfer ζ to $T_{\zeta \to P\eta}^{\frac{1}{q}}\zeta$. q groups of such control inputs together with the control inputs that transfer $P\xi$ to ζ are the desired ones which steer the nearly-controllable system (1) from ξ to η.

As we can see from Algorithm 1, there are four parameters $\lambda_{m+1}, \lambda_{m+2}, q$ and K which influence the control inputs for the system to achieve state transition and there exist infinite groups of such control inputs. This is different from the diagonal case studied in [13,14] that there is only one parameter K which influences the control inputs. Therefore, it is much more complicated to find the minimum-energy control inputs in the nondiagonalizable case. This will be seen more clearly when we derive the energy function in the next section.

Through the analysis on Algorithm 1, we finish the programming of each algorithm module and preliminary improvement with it, including the solution and verification of control inputs. Note that Algorithm 1 can be completed in polynomial time with computational time complexity $O(n^2)$.

That is, as the input increases, the running time of Algorithm 1 increases in polynomial form rather than exponential form. Therefore, the algorithm can work efficiently even if considering high-dimensional system (1).

Since Algorithm 1 can offer the control sequence but not an optimal one, in this paper, we propose a new algorithm to try to find a minimum-energy control sequence. To make the idea more clear, we focus on the case when the initial state ξ and terminal state η belong to the same orthant of $\mathbb{R}^n$, i.e., ξ_i and η_i have the same sign for $i = 2, 4, \ldots, 2r, 2r+1, \ldots, n$.

The minimum-energy control problem for the nearly-controllable system (1) with finite control inputs is stated as follows: For any given initial state ξ and any given terminal state η in the same orthant of $\mathbb{R}^n$, define the control energy by

$$W = q \times \left(\sum_{i=0}^{2m+2} u^2(i) \right), \tag{5}$$

where $u(0), u(1), \ldots, u(2m+2)$ are the control inputs that steer the system from ξ to $T_{\xi\to\eta}^{\frac{1}{q}}\xi$, then choose suitable variables to minimize W.

Remark 1. If B in system (1) is diagonalizable, i.e. $r = 0$, the minimum-energy control problem has been studied in [13] and the minimum-energy control sequence can be computed by applying the algorithm proposed in [13]. We only need to let A be an identity matrix in [13], and then obtain the following analytic expression for the control energy from [13], which is

$$\begin{aligned} W &= W(K) = (\phi_n^2 - 2\phi_{n-1}) \\ &\quad - \frac{2\prod_{i=1}^{n}\lambda_i(\phi_n + \sum_{i=1}^{n}\frac{1}{\lambda_i})}{K} + \frac{\prod_{i=1}^{n}(\lambda_i)^2}{K^2} \end{aligned}$$

defined on $(-\infty, 0) \cup (0, +\infty)$, where ϕ_n, ϕ_{n-1} and λ_i are as given in Theorem 1 in [13]. Thus, K is the only parameter so that the minimum-energy of system (1) can be easily analyzed by derivating $W(K)$ with respect to K. Furthermore, [13] proposed an algorithm to find the K and corresponding minimum value of $W(K)$. But for system (1) with nondiagonalizable structure, there are four parameters λ_{m+1}, λ_{m+2}, q and K to choose and W becomes a function of four variables. The method in [13] is no longer applicable.

III. MAIN RESULTS

In this section, we propose an algorithm to solve the minimum-energy control problem of the nearly-controllable system (1). To this end, we need to derive the control energy W.

Note that $1 + KG(s) = 0 \Leftrightarrow$

$$\begin{aligned} &s(s+\lambda_1)^2 \cdots (s+\lambda_m)^2 (s+\lambda_{m+1})(s+\lambda_{m+2}) + \\ &K\left((-1)^{2m+2}\mu_1 s^{2m+2} + \cdots + (-1)\mu_{2m+2}s + 1\right) = 0. \end{aligned}$$

Since the above equation has no zero root if $K \neq 0$, multiplying both sides of it by $\frac{1}{s^{2m+3}}$, we have

$$\begin{aligned} &\left(1+\frac{\lambda_1}{s}\right)^2 \cdots \left(1+\frac{\lambda_m}{s}\right)^2 \left(1+\frac{\lambda_{m+1}}{s}\right)\left(1+\frac{\lambda_{m+2}}{s}\right) + \\ &K\left((-1)^{2m+2}\mu_1\frac{1}{s} + \cdots + (-1)\mu_{2m+2}\frac{1}{s^{2m+2}} + \frac{1}{s^{2m+3}}\right) \\ &= 0. \end{aligned} \tag{6}$$

Let $z = \frac{1}{s}$, then eq. (6) can be rewritten as

$$\begin{aligned} &(1+\lambda_1 z)^2 \cdots (1+\lambda_m z)^2 (1+\lambda_{m+1}z)(1+\lambda_{m+2}z) + \\ &K\left((-1)^{2m+2}\mu_1 z + \cdots + (-1)\mu_{2m+2}z^{2m+2} + z^{2m+3}\right) \\ &= 0. \end{aligned} \tag{7}$$

where the coefficients of z^{2m+3}, z^{2m+2}, and z^{2m+1} are

$$\begin{aligned} &K, \left(K(-1)\mu_{2m+2} + \lambda_{m+1}\lambda_{m+2}\prod_{i=1}^{m}\lambda_i^2\right), \\ &K(-1)^2\mu_{2m+1} + \lambda_{m+1}\lambda_{m+2}\times \\ &\prod_{i=1}^{m}\lambda_i^2\left(\frac{1}{\lambda_{2m+1}} + \frac{1}{\lambda_{2m+2}} + \sum_{i=1}^{m}\frac{2}{\lambda_i}\right) \end{aligned}$$

respectively. It follows from the Viète's formulas that

$$\begin{gathered} \sum_{i=1}^{2m+3} z_i = \mu_{2m+2} - \frac{\lambda_{m+1}\lambda_{m+2}\prod_{i=1}^{m}\lambda_i^2}{K}, \\ \sum_{1\le i_1<i_2\le 2m+3} z_{i_1}z_{i_2} = \mu_{2m+1} + \\ \frac{\lambda_{m+1}\lambda_{m+2}\prod_{i=1}^{m}\lambda_i^2\left(\frac{1}{\lambda_{2m+1}} + \frac{1}{\lambda_{2m+2}} + \sum_{i=1}^{m}\frac{2}{\lambda_i}\right)}{K}, \end{gathered}$$

where $z_1, z_2, \ldots, z_{2m+3}$ are the roots of (7). In view of (5), we obtain

$$\begin{aligned} W &= q \times \left(\sum_{i=0}^{2m+2} u^2(i)\right) \\ &= q \times \left(\sum_{i=1}^{2m+3}\frac{1}{s^2(i)}\right) = q \times \left(\sum_{i=1}^{2m+3} z_i^2\right) \\ &= q \times \left[\left(\sum_{i=1}^{2m+3} z_i\right)^2 - 2\sum_{1\le i_1<i_2\le 2m+3} z_{i_1}z_{i_2}\right] \\ &= q \times \left(\mu_{2m+2} - \frac{\lambda_{m+1}\lambda_{m+2}\prod_{i=1}^{m}\lambda_i^2}{K}\right)^2 - 2q \times \mu_{2m+1} \\ &\quad - \frac{2q \times \lambda_{m+1}\lambda_{m+2}\prod_{i=1}^{m}\lambda_i^2 \cdot \left(\frac{1}{\lambda_{2m+1}} + \frac{1}{\lambda_{2m+2}} + \sum_{i=1}^{m}\frac{2}{\lambda_i}\right)}{K}. \end{aligned}$$

That is, we can obtain the analytical expression of the energy function in the nondiagonalizable structure case, which is

$$W(\lambda_{m+1},\lambda_{m+2},q,K)=q\times(\mu_{2m+2}^2-2\mu_{2m+1})-\frac{2q\lambda_{m+1}\lambda_{m+2}\prod_{i=1}^{m}\lambda_i^2\left(\mu_{2m+2}+\frac{1}{\lambda_{2m+1}}+\frac{1}{\lambda_{2m+2}}+\sum_{i=1}^{m}\frac{2}{\lambda_i}\right)}{K}+\frac{q\left(\lambda_{m+1}\lambda_{m+2}\prod_{i=1}^{m}\lambda_i^2\right)^2}{K^2}, \tag{8}$$

where μ_{2m+1}, μ_{2m+2} are related to $\lambda_{m+1},\lambda_{m+2}$ and q in (4). According to (8), if λ_{m+1}, λ_{m+2} and q are fixed, we can always minimize the control energy $W(K)$ by applying the algorithms proposed in [13]. As a result, our goal is to configure control variables λ_{m+1}, λ_{m+2} and q to minimize the control energy $W(\lambda_{m+1},\lambda_{m+2},q)$.

Although we have derived the analytical expression of W, it is not an analytical task to minimize it. This is because if we take $\lambda_{m+1},\lambda_{m+2},q$ as parameters not fix numbers, then μ_{2m+1} and μ_{2m+2}, which are both functions with respect to them, cannot be analytically represented. Therefore, to analyze W, we need to turn to an experimental way and apply the root locus approach by Matlab. From (5), we know that W is also determined by control inputs $u(i)$, which are reflected as the reciprocals of the root on the root locus. Therefore, the closer the root is to the origin, the greater W is. Based on the above analysis and the fact that $|\lambda_{m+1}|$ is the minimum in (3), we can roughly conclude that the control energy is mainly determined by two factors: the root locus starting from the origin denoted as α and the root locus starting from λ_{m+1} denoted as β. If there are no other restrictions, we only need to make α far enough away from the origin (i.e. at the breakaway point), and by setting the appropriate λ_{m+1} to make root of β the maximum modulus (i.e. at the saddle point), the control energy can be minimized.

Therefore, we propose a parameter configuration method based on experiment and root locus theory so that the control energy can be obtained which is very close to the ideal minimum energy. Compared to the algorithms in the diagonalizable structure case of [13], $\lambda_{m+1},\lambda_{m+2},q$ are also considered in the analysis of the minimum-energy problem as control variables and thus the algorithm is more general.

Algorithm 2. Given a nearly-controllable system as in (1) with initial state ξ and terminal state η in the same orthant of $\mathbb{R}^n$. Configure $\lambda_{m+1},\lambda_{m+2},q,K$ and find the minimum-energy control to steer the system from ξ to η.

Step 1. Apply Algorithm 1 to get the transfer function and the corresponding root loci. Let γ denote the root locus starting from λ_{m+2} and δ denote the root locus with the largest gain of the breakaway point except β. To analyze q, first choose a certain value of q and configure $\lambda_{m+1},\lambda_{m+2}$.

Step 2. First select the appropriate λ_{m+1} and fix K at the breakaway point of δ. There are following three situations of possible minimum control energy with $|\lambda_{m+1}|$ decrease: (i) β is between the pole and the breakaway point; (ii) β is exactly at the saddle point; (iii) β is between the saddle point and the zero. Judge whether α is exactly δ. If so, adjust the value of $|\lambda_{m+1}|$ such that the point of β is exactly at the saddle point, i.e. being situation (ii). Otherwise α is not at the breakaway point in situation (ii), so adjust the value of $|\lambda_{m+1}|$ at situation (iii) such that the control energy is minimum.

Step 3. Choose some test groups of λ_{m+2} in a certain range and get the corresponding λ_{m+1} and the control energy by skipping back to Step 2. If γ enters the complex domain at this time, the minimum energy and the set of parameters needs to be omitted, which is unavailable. Compare the control energy of every group with each other and obtain a group with minimum control energy as well as λ_{m+2}. Hence λ_{m+1} and λ_{m+2} are configured accurately when q is fixed. But the control energy is not necessarily the minimum in the case of fixing K at the breakaway point of δ, so K needs to be reconfigured.

Step 4. Consider (8) as a function of one parameter, i.e. $W(K)$, hence we can apply Algorithm 1 in [13] to get an extreme value. Then get the minimum-energy control $W(K)$ and K accordingly.

Step 5. Back to Step 1, change the value of q and repeat the previous steps. Owing to that larger value of q makes larger control energy when the linear term q in (5) dominates, only a few groups of q are needed to get the results. Compute groups of control energy under different q by taking points of root loci experimentally, and finally select the minimum control energy and corresponding q.

Remark 2. λ_{m+1} is directly related to the root locus β. That is, when $|\lambda_{m+1}|$ is too large or too small, the shape of β change a lot. When $|\lambda_{m+1}|$ is too small, the saddle point vanishes so that the gain can only be chosen at the breakaway point of β, which makes the root on α close to the origin. The first factor α plays a major role and the control energy is very large; when $|\lambda_{m+1}|$ is too large, the root on β is very close to the origin, and the second factor β plays a major role, so the control energy is still very large. Therefore, when $|\lambda_{m+1}|$ decrease bit by bit, there exists a lower bound. Apart from that, we note a restriction that the roots on α and β in the complex domain cannot be selected. Therefore, we can always find groups of λ_{m+1} and K to satisfy the control energy within a certain range, like situation (ii) in Step 2.

Remark 3. Step 5 of configuring q is more tedious in taking points, which takes a lot of time. We have to choose many test groups of q, and get corresponding λ_{m+1} and λ_{m+2}. In fact, we do not need to pre-configure λ_{m+1} and λ_{m+2} every time by simplifying the Algorithm 2, but fix the parameter λ_{m+2}. Each test group of q only needs to fix λ_{m+2} and configure different λ_{m+1} to get the control energy of the system (1), then we can determine the value of q when the energy is minimum. Finally, the minimum energy control of the system (1) can be roughly obtained by configuring λ_{m+1} and λ_{m+2} directly after q has been fixed. Suppose m test groups of $\lambda_{m+1}, \lambda_{m+2}$ in Step 3 and n test groups of q in Step 5, then we only need to compute $(m+n)$ times of control energy rather than $(m \times n)$ times, which greatly reduces the complexity. Two examples in the following section both adopt the simplification method.

IV. SIMULATIONS

In this section, we present two examples to illustrate Algorithm 2.

Example 1. Consider the system

$$x(k+1) = \left(I + u(k) \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1.8 & 1 & 0 \\ 0 & 0 & 0 & 1.8 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix} \right) x(k), \tag{9}$$

with initial state $\xi = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T$ and terminal state $\eta = \begin{bmatrix} 20 & 20 & 20 & 10 & 10 \end{bmatrix}^T$, where $x(k) \in \mathbb{R}^5$, $u(k) \in \mathbb{R}$ and $m = 3$. According to Theorem 1, there exist q groups of control inputs steering the system from ξ to η, each group of which contains 9 control inputs. The reciprocals of the 9 control inputs are on the root loci of $1 + KG(s) = 0$ in [4]. The system (9) has four pairs of root loci, and α is not δ, i.e. the root locus with the largest gain of the breakaway point. Therefore, we adjust the value of $|\lambda_4|$ at the situation (iii) in Step 2 of Algorithm 2. According to Remark 3, we fix λ_5 a appropriate value, select different groups of q and compute the minimum energy in Algorithm 2. For example, we take $\lambda_5 = -3$, and adjust value of λ_4 to balance root of α and root of β. Groups of control energy and λ_4 are shown in Fig. 1.

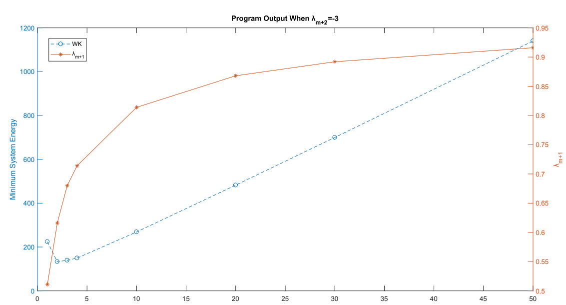

Fig. 1. The diagram of $\lambda_5 = -3$

In Fig. 1, blue points denote the minimum control energy and red points denote the corresponding $|\lambda_4|$ when q varies. It should be noted that the "minimum" here is for λ_4. Since minimum control energy has an extreme value when $q = 2$, we can configure $q = 2$ and obtain λ_4, λ_5 according to Steps 2, 3. Groups of control energy and λ_4 are shown in Fig. 2.

In Fig. 2, blue points denote the minimum control energy and red points denote the corresponding $|\lambda_4|$ when λ_5 varies. The "minimum" here is also for λ_4. More specifically, the black points need to be excluded, which are the unsolvable

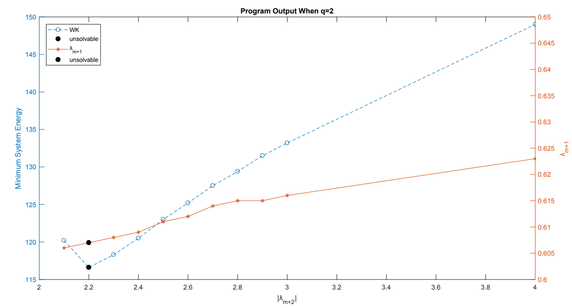

Fig. 2. The diagram of $q = 2$

points in Step 3 (γ enters the complex domain). From Fig. 2 we know that the minimum control energy takes in the case that $q = 2$, $\lambda_4 = 0.608$, $\lambda_5 = -2.3$. By choosing $K = 1.0454$ in Step 4, the corresponding root loci are shown in Fig. 3.

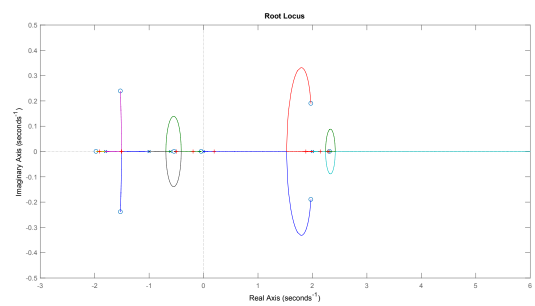

Fig. 3. The root loci of $1 + KG(s) = 0$

From the root loci of $1 + KG(s) = 0$ in Fig. 3, we obtain the roots and compute the corresponding reciprocals, which are

$$\begin{aligned} u(0) &\approx 0.4372,\ u(1) \approx 0.4667,\ u(2) \approx 0.5323, \\ u(3) &\approx -0.5232,\ u(4) \approx -0.6624,\ u(5) \approx -0.6652, \\ u(6) &\approx -1.9998,\ u(7) \approx 5.1406,\ u(8) \approx -5.1846. \end{aligned}$$

One can now verify that, by 2 groups of the above control inputs, system (9) can be steered from ξ to η and the minimum control energy

$$W = 2 \times \left(\sum_{i=0}^{8} u^2(i) \right) \approx 118.3.$$

Example 2. Consider the system

$$x(k+1) = \left(I + u(k) \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix} \right) x(k), \tag{10}$$

with initial state $\xi = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$ and terminal state $\eta = \begin{bmatrix} 20 & 20 & 10 & 10 \end{bmatrix}^T$, where $x(k) \in \mathbb{R}^4$, $u(k) \in \mathbb{R}$ and $m = 2$. According to Theorem 1, each group exists 7 control inputs. Compared with Example 1, α is the root locus δ exactly. Therefore, we turn to adjust the value of $|\lambda_3|$ at the situation (ii) in Step 2 of Algorithm 2. Also let $\lambda_4 = -3$, then we compute the minimum energy and determine corresponding q. Groups of control energy and λ_3 are shown in Fig. 4.

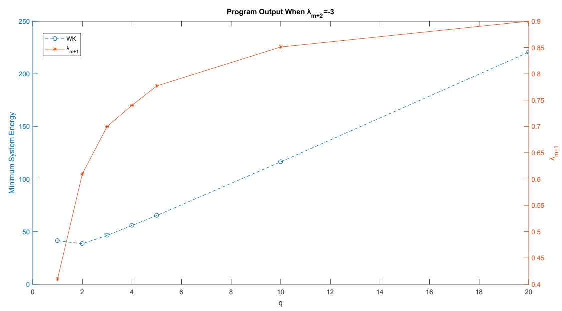

Fig. 4. The diagram of $\lambda_4 = -3$

From above diagram, we configure $q = 2$ and obtain λ_3, λ_4 according to Steps 2, 3. Groups of control energy and λ_3 are shown in Fig. 5.

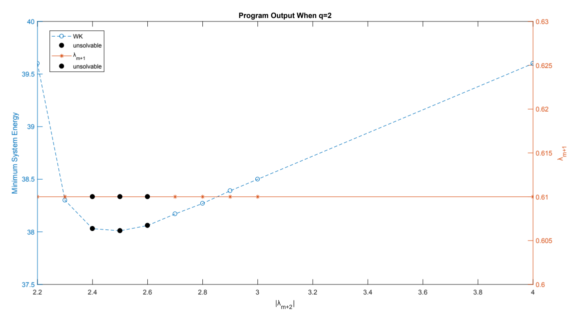

Fig. 5. The diagram of $q = 2$

From Fig. 5 we finally obtain that the minimum control energy takes in the case that $q = 2$, $\lambda_3 = 0.61$, $\lambda_4 = -2.7$. Choose $K = 1.0454$ by Step 4, and the corresponding root loci are shown in Fig. 6. Computing the reciprocals of the roots yields

$$u(0) \approx 0.3763,\ u(1) \approx 0.4127,\ u(2) \approx 0.6719,$$
$$u(3) \approx -0.8237,\ u(4) \approx 1.4271,\ u(5) \approx -2.7929,$$
$$u(6) \approx -2.7929.$$

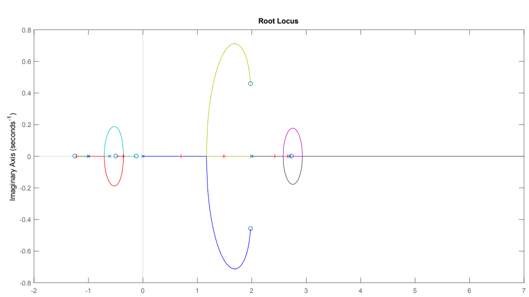

Fig. 6. The root loci of $1 + KG(s) = 0$

By 2 groups of the above control inputs, system (10) can be steered from ξ to η and the minimum control energy

$$W = 2 \times \left(\sum_{i=0}^{6} u^2(i) \right) \approx 38.17.$$

V. CONCLUSIONS

In this paper, the problem of minimum-energy control for a class of nearly-controllable discrete-time bilinear systems is considered. Based on Matlab, a parameter configuration algorithm is proposed to solve the minimum-energy control inputs to steer the systems between a given pair of states. Examples are given to illustrate the proposed algorithm which show that the minimum-energy is solvable by experimental method.

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